Abelian hypermultiplet gaugings and BPS vacua in N = 2 supergravity

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Based on [1409.6310, H. E.–Halmagyi].

Outline

Outline: 1. Introduction

Motivations

Asymptotically AdS_4 BPS black holes (BH) are very important:

- BH entropy computations $\rightarrow$ need near-horizon geometries
- String theory and M-theory embeddings
- AdS/CFT correspondence

Black hole: interpolation

$$\text{magnetic } \mathrm{AdS} \text{ (UV)} \rightarrow \text{near-horizon geometry (IR)}$$

AdS_4 and near-horizon geometry $\rightarrow$ supergravity solutions

BPS equations for AdS_4 vacua

BPS equations reduces to

$$\langle \mathcal{P}, \mathcal{V} \rangle \propto \frac{1}{R_{\text{AdS}}}, \qquad \langle \mathcal{K}^u, \mathcal{V} \rangle = 0$$

$\mathcal{P}$ moment map, $\mathcal{K}^u$ Killing vectors, $\mathcal{V}$ symplectic section

But

$$\mathcal{P} = \omega_u^3 \mathcal{K}^u \Longrightarrow \langle \mathcal{P}, \mathcal{V} \rangle = 0.$$

→ no regular solution [0911.2708, Cassani et al.][1204.3893, Louis et al.]

Missing piece

$$\mathcal{P} = \omega_u^3 \mathcal{K}^u + \mathcal{W}$$

→ need to understand (special and) quaternionic isometries
[de Wit–van Proeyen '90] [hep-th/9210068, de Wit–Vanderseypen–van Proeyen] [hep-th/9310067, de Wit–van Proeyen]

Outline: 2. $N = 2$ supergravity

$N = 2$ supergravity: multiplets

[hep-th/9605032, Andrianopoli et al.] [*Supergravity*, Freedman–van Proeyen]

- Gravity multiplet
$$\{g_{\mu\nu}, \psi^{\alpha}_{\mu}, A^{0}_{\mu}\}$$
$\alpha = 1, 2$
- n_v vector multiplets
$$\{A^{i}_{\mu}, \lambda^{\alpha i}, \tau^{i}\}$$
$i = 1, \ldots, n_v$
- n_h hypermultiplets
$$\{\zeta^{\mathcal{A}}, q^{u}\}$$
$u = 1, \ldots, 4n_h, \qquad \mathcal{A} = 1, \ldots, 2n_h$

$N = 2$ supergravity: bosonic lagrangian

$$\mathcal{L}_{\text{bos}} = \frac{R}{2} + \frac{1}{4}\,\text{Im}\,\mathcal{N}(\tau)_{\Lambda\Sigma}\,F^{\Lambda}_{\mu\nu}\,F^{\Sigma\mu\nu} - \frac{1}{8}\,\text{Re}\,\mathcal{N}(\tau)_{\Lambda\Sigma}\,\varepsilon^{\mu\nu\rho\sigma}F^{\Lambda}_{\mu\nu}\,F^{\Sigma}_{\rho\sigma}$$
$$- g_{i\bar{\jmath}}(\tau)\,\partial_\mu\tau^i\partial^\mu\bar{\tau}^{\bar{\jmath}} - \frac{1}{2}\,h_{uv}(q)\,\partial_\mu q^u\partial^\mu q^v$$

Field strengths

$$F^\Lambda = \mathrm{d}A^\Lambda, \qquad A^\Lambda = (A^0, A^i)$$

$\Lambda = 0, \ldots, n_v$

Electromagnetic duality

Dual (magnetic) field strengths

$$G_\Lambda = \star\left(\frac{\delta\mathcal{L}_{\text{bos}}}{\delta F^\Lambda}\right) = \text{Re}\,\mathcal{N}_{\Lambda\Sigma}\,F^\Lambda + \text{Im}\,\mathcal{N}_{\Lambda\Sigma}\,\star F^\Lambda$$

Maxwell equations and Bianchi identities

$$\mathrm{d}F^\Lambda = 0, \qquad \mathrm{d}G_\Lambda = 0$$

invariant under symplectic transformations

$$\begin{pmatrix} F^\Lambda \\ G_\Lambda \end{pmatrix} \longrightarrow \mathcal{U}\begin{pmatrix} F^\Lambda \\ G_\Lambda \end{pmatrix}, \qquad \mathcal{U} \in \mathrm{Sp}(2n_v+2,\mathbb{R})$$

The action is **not invariant**

Scalar geometry

Non-linear sigma model: scalar fields $\rightarrow$ coordinates on target space

$$\mathcal{M} = \mathcal{M}_v(\tau^i) \times \mathcal{M}_h(q^u)$$

with

- $\mathcal{M}_v$ special Kähler manifold, $\dim_{\mathbb{R}} = 2n_v$
- $\mathcal{M}_h$ quaternionic Kähler manifold, $\dim_{\mathbb{R}} = 4n_h$

Isometry group (global symmetries)

$$G \equiv \mathrm{ISO}(\mathcal{M}), \qquad G \subset \mathrm{Sp}(2n_v + 2)$$

[hep-th/9605032, Andrianopoli et al.]

Gaugings: general case

Local gauge group H

$$H \subset G$$

encoded by Killing vectors $\{k^i_\Lambda, k^u_\Lambda\}$

Covariant derivatives on scalars (minimal coupling)

$$\partial_\mu \longrightarrow \mathrm{D}_\mu = \partial_\mu - A^\Lambda_\mu \left[k^i_\Lambda(\tau) \frac{\partial}{\partial \tau^i} + k^u_\Lambda(q) \frac{\partial}{\partial q^u} \right]$$

Generates scalar potential $V(\tau, q) \to \mathrm{AdS}_4$ vacua

Outline: 3. Kähler geometries

Kähler manifold: definition

Manifold (M, g) with

- Hermitian metric

$$\mathrm{d}s^2 = 2\, g_{i\bar{\jmath}}\, \mathrm{d}\tau^i \mathrm{d}\bar{\tau}^{\bar{\jmath}}, \qquad i = 1, \ldots, n$$

- Complex structure

$$J^2 = -1, \qquad J\, g\, J^t = g$$

- Fundamental 2-form

$$\Omega = -2\, J_{i\bar{\jmath}}\, \mathrm{d}\tau^i \wedge \mathrm{d}\bar{\tau}^{\bar{\jmath}}, \qquad \mathrm{vol} = \Omega^n$$

- Kähler condition

$$\mathrm{d}\Omega = 0$$

Kähler manifold: Kähler potential

- Metric given by Kähler potential $K(\tau,\bar{\tau})$

$$g_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} K, \qquad \partial_i \equiv \frac{\partial}{\partial \tau^i}$$

- Metric invariant under Kähler transformations

$$K(\tau,\bar{\tau}) \longrightarrow K(\tau,\bar{\tau}) + f(\tau) + \bar{f}(\bar{\tau})$$

Special Kähler manifold: definition

- Kähler manifold with bundle $\mathrm{Sp}(2n_v+2,\mathbb{R})$ and section

$$v = \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}, \qquad F_\Lambda = \frac{\partial F}{\partial X^\Lambda}$$

  (assuming F exists)
- Prepotential F

$$F(\lambda X) = \lambda^2 F(X)$$

- Homogeneous coordinates X^Λ, special coordinates

$$\tau^i = \frac{X^i}{X^0}$$

  (common choice: $X^0 = 1$)

Special Kähler manifold: symplectic structure

- Symplectic inner product

$$\langle A, B\rangle = A^t \Omega B = A^\Lambda B_\Lambda - A_\Lambda B^\Lambda, \qquad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- Kähler potential

$$\mathrm{e}^{-K} = -i\,\langle v, \bar{v}\rangle = -i(X^\Lambda \bar{F}_\Lambda - F_\Lambda \bar{X}^\Lambda)$$

- Covariant section

$$\mathcal{V} = \mathrm{e}^{\frac{K}{2}} v = \mathrm{e}^{\frac{K}{2}} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}$$

- Covariant Kähler derivative

$$U_i \equiv \mathrm{D}_i \mathcal{V} = \left(\partial_i + \frac{1}{2}\,\partial_i K\right) \mathcal{V}$$

Special Kähler manifold: structure

- Gauge coupling matrix $\mathcal{N}$ (built from F)

$$F_\Lambda = \mathcal{N}_{\Lambda\Sigma} X^\Sigma$$

- Complex structure $\mathcal{M}$ on the bundle (built from $\mathcal{N}$)

$$\mathcal{M}\mathcal{V} = -i\,\mathcal{V}, \qquad \mathcal{M}\mathrm{D}_i\mathcal{V} = i\,\mathrm{D}_i\mathcal{V}$$

Special Kähler manifold: prepotentials

Usual examples and Calabi–Yau have:

- cubic

$$F = -D_{ijk}\,\frac{X^i X^j X^k}{X^0}$$

- quadratic

$$F = \frac{i}{2}\,\eta_{\Lambda\Sigma}\,X^\Lambda X^\Sigma$$

$\eta = \mathrm{diag}(-1, 1, \ldots, 1)$

Quaternionic manifold: definition

- Metric

$$\mathrm{d}s^2 = h_{uv}\,\mathrm{d}q^u \mathrm{d}q^v, \qquad u = 1, \ldots, 4n_h$$

holonomy $\mathrm{SU}(2) \times \mathrm{Sp}(n_h)$

- Complex structure triplet $J^x, x = 1, 2, 3$

$$\forall x : \qquad J^x h (J^x)^t = h$$

- $\mathrm{SU}(2)$ algebra

$$J^x J^y = -\delta^{xy} + \varepsilon^{xyz} J^z$$

- Hyperkähler forms

$$K^x = J^x_{uv}\,\mathrm{d}q^u \wedge \mathrm{d}q^v$$

Quaternionic manifold: $\mathrm{SU}(2)$ properties

- $\mathrm{SU}(2)$ connection $\omega^x = \omega^x_u \mathrm{d}q^u$
- Curvature 2-form

$$\Omega^x = \nabla \omega^x = \mathrm{d}\omega^x + \frac{1}{2}\,\varepsilon^{xyz}\omega^y \wedge \omega^z$$

with

$$\Omega^x = \lambda K^x, \qquad \lambda \in \mathbb{R}$$

Supersymmetry $\to \lambda = -1$

- Fundamental 4-form

$$\varOmega = \Omega^x \wedge \Omega^x, \qquad \mathrm{d}\varOmega = 0, \qquad \mathrm{vol} = \varOmega^n$$

Quaternionic manifold: c-map construction

$$\mathrm{d}s^2 = \mathrm{d}\phi^2 + 2\, g_{a\bar{b}} \mathrm{d}z^a \mathrm{d}\bar{z}^{\bar{b}} + \frac{\mathrm{e}^{4\phi}}{4}\left(\mathrm{d}\sigma + \frac{1}{2}\,\xi^t \mathbb{C} \mathrm{d}\xi\right)^2 - \frac{\mathrm{e}^{2\phi}}{4}\, \mathrm{d}\xi^t \mathbb{C}\mathbb{M} \mathrm{d}\xi$$

$a = 1, \dots, n_h - 1, \qquad A = (0, a)$

- Heisenberg fibers:
  - Dilaton ϕ, axion σ, Ramond pseudo-scalars $\xi = (\xi^A, \tilde{\xi}_A)$
- Base special Kähler $\mathcal{M}_z$:
  - Prepotential G, Kähler potential K_Ω, metric $g_{a\bar{b}}$
  - Symplectic vector $Z = (Z^A, G_A)$
  - $\mathbb{C}$ symplectic metric, $\mathbb{M}$ complex structure

Outline: 4. Kähler isometries

Killing vectors and Lie derivative

- Isometry of spacetime (M, g): transformation that preserved distance (i.e. the metric)
- Acts with Lie derivative, generated by Killing vector k

$$\mathcal{L}_k g = 0$$

- Set of Killing vectors $\rightarrow$ Lie algebra

$$[k_a, k_b] = f_{ab}{}^c k_c, \qquad a = 1, \ldots, \dim \mathrm{ISO}(M)$$

Kähler manifolds: moment maps

- Holomorphic Killing vectors → preserve complex structure

$$\mathcal{L}_k J = 0$$

- Killing vectors given by moment maps

$$k_i = i\,\partial_i P, \qquad k_{\bar{\imath}} = -i\,\partial_{\bar{\imath}} P$$

  $P(\tau^i, \bar{\tau}^{\bar{\imath}}) \in \mathbb{R}$
  Gives the coupling of the gravitini ψ_μ^α to A_μ^Λ

- Kähler potential invariant up to Kähler transformation

$$\mathcal{L}_k K = 2\,\mathrm{Re}\, f$$

Quaternionic manifolds: triholomorphic isometries

- A transformation may induce a change of basis for J^x

$$\mathcal{L}_k \Omega^x = \varepsilon^{xyz}\, W_k^y\, \Omega^z$$

(recall $\Omega^x = \lambda K^x$, K^x defined from J^x)
W_k called $\mathrm{SU}(2)$ compensator ($=$ rotation vector)

- Connection

$$\mathcal{L}_k \omega^x = \nabla\, W_k^x$$

$\rightarrow$ use this formula to compute W_k

- C-map isometry

$$g \in \mathrm{ISO}(\mathcal{M}_h) \Longrightarrow g|_{\mathcal{M}_z} \in \mathrm{ISO}(\mathcal{M}_z)$$

- Triholomorphic moment maps

$$i_k \Omega^x = -\nabla\, P^x \Longrightarrow P_k^x = i_k \omega^x + W_k^x$$

Special Kähler manifolds: isometries

- For $\mathcal{U} = \mathrm{e}^{\mathfrak{U}} \in \mathrm{Sp}(2n_v + 2)$

$$\begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} \to \begin{pmatrix} X'^\Lambda \\ F'_\Lambda \end{pmatrix} = \mathcal{U} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}, \qquad \delta \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} = \mathfrak{U} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}$$

  $\to$ new prepotential $F'(X')$

- $\mathcal{U} \subset \mathrm{ISO}(\mathcal{M}_v) \to$ symmetries of the action

$$F'(X') = F(X') \Longleftrightarrow X^\Lambda \, \delta F_\Lambda = F_\Lambda \, \delta X^\Lambda$$

  $\to$ conditions on $\mathcal{U}$ (or $\mathfrak{U}$)

Special Kähler isometries: cubic prepotential

- Recall:

$$F = -D_{ijk}\,\frac{X^i X^j X^k}{X^0}$$

- Isometries parametrized by $\{\beta, b^i, a_i, B^i{}_j\}$

$$\delta\tau^i = b^i - \frac{2}{3}\,\beta\,\tau^i + B^i{}_j\tau^j - \frac{1}{2}\,a_\ell\,R^i{}_{jk}{}^\ell\,\tau^j\tau^k$$

Constraints on $a_i, B^i{}_j$

- Kähler transformation

$$\mathcal{L}_k K = 2\,\mathrm{Re}(\beta + a_i\tau^i)$$

- Quartic invariant

$$\mathcal{I}_4(p,q) = -\,(q_\Lambda p^\Lambda)^2 + \frac{1}{16}\,p^0\,\hat{D}^{ijk}q_i q_j q_k - 4\,q_0\,D_{ijk}p^i p^j p^k + \frac{9}{16}\,\hat{D}^{ijk}D_{k\ell m}q_i q_j\,p^\ell p^m$$

Special Kähler isometries: quadratic prepotential

- Recall:

$$F = \frac{i}{2}\,\eta_{\Lambda\Sigma}\,X^\Lambda X^\Sigma$$

- Isometries parametrized by $A^\Lambda{}_\Sigma$ (with constraints)

$$\delta\tau^i = A^i{}_0 + (A^i{}_j - A^0{}_0\delta^i{}_j)\tau^j - A^0{}_j\tau^i\tau^j$$

- Kähler transformation

$$\mathcal{L}_k K = 4\,\mathrm{Re}(\overline{A}^i{}_0\tau^i)$$

- Quartic invariant

$$\mathcal{I}_4(p,q) = -\frac{1}{4}\left(p^\Lambda\eta_{\Lambda\Sigma}\,p^\Sigma + q_\Lambda\eta^{\Lambda\Sigma}q_\Sigma\right)^2$$

Quaternionic isometries: fiber and duality symmetries

- Transformations of the fiber, model-independent

$$h_+ = \partial_\sigma, \quad h_\alpha = \mathbb{C}\partial_\xi + \xi\,\partial_\sigma,$$
$$h_0 = \partial_\phi - 2\sigma\partial_\sigma - \xi\,\partial_\xi$$

  Translations and scalings. h_α is $2n_h$ vectors

- Transformations lifted from $\mathcal{M}_z$, model-dependent

$$h_{\mathbb{U}} = (\mathbb{U}Z)^t\partial_Z + \text{c.c.} + (\mathbb{U}\xi)^t\partial_\xi$$

  $\mathbb{U} \in \mathfrak{sp}(2n_h)$, constant parameters
  Transformation on ξ compensates the one on z^a

Quaternionic isometries: hidden symmetries 1

Metric on $\mathcal{M}_z$ invariant only for $\mathbb{U} \in \mathfrak{iso}(\mathcal{M}_z) \subset \mathfrak{sp}(2n_h)$

Idea for new symmetries

Promotes $\mathbb{U}$ to a field-dependent matrix $\mathbb{S}$.

Hidden vectors $k_-, k_{\hat\alpha}$, model dependent

$$\delta_{k_-} Z = \mathbb{S} Z, \qquad \delta_{k_{\hat\alpha}} Z = \mathbb{C} \partial_\xi \mathbb{S} Z$$

where

$$\mathbb{S} = \frac{1}{2} \left(\xi \xi^t + \frac{1}{2} \mathbb{C} \partial_\xi \big(\mathbb{C} \partial_\xi \mathcal{I}_4(\xi) \big)^t \right)$$

$\mathcal{I}_4(\xi)$ well defined only for symmetric $\mathcal{M}_h$ [0902.3973, Cerchiai–Ferrara–Marrani–Zumino]

Quaternionic isometries: hidden symmetries 2

Explicit formulas

$$
\begin{aligned}
h_- =& -\sigma\partial_\phi + (\sigma^2 - \mathrm{e}^{-4\phi} - W)\partial_\sigma + (\sigma\xi - \mathbb{C}\partial_\xi W)^t\partial_\xi \\
& - (\mathbb{S}Z)^t\partial_Z + \mathrm{c.c.}, \\
h_{\hat\alpha} =& -\frac{1}{2}\,\xi\,\partial_\phi + \left(\frac{\sigma}{2}\,\xi - \frac{1}{2}\,\mathbb{C}\partial_\xi W\right)\partial_\sigma + \sigma\,\mathbb{C}\partial_\xi \\
& + \left(\frac{1}{2}\,\xi\xi^t - \mathbb{C}\partial_\xi(\mathbb{C}\partial_\xi W)^t\right)\partial_\xi - (\mathbb{C}\partial_\xi\mathbb{S}\,Z)^t\partial_Z + \mathrm{c.c.}
\end{aligned}
$$

where

$$
\begin{aligned}
W &= \frac{1}{4}\,\mathcal{I}_4(\xi^A,\tilde\xi_A) - \frac{1}{2}\,\mathrm{e}^{-2\phi}\,\xi^t\mathbb{C}\mathbb{M}\xi, \\
\mathbb{S} &= \frac{1}{2}\left(\xi\xi^t + \frac{1}{2}\,H\right)\mathbb{C}, \\
H &= \mathbb{C}\partial_\xi(\mathbb{C}\partial_\xi\mathcal{I}_4(\xi))^t
\end{aligned}
$$

Quaternionic isometries: compensators

- Define

$$P^{\pm} = P^1 \pm i\, P^2, \qquad W^{\pm} = W^1 \pm i\, W^2$$

- Computations done in special coordinates
  (since ω^x invariant in homogeneous coordinates)
- Duality symmetries

$$\text{cubic:} \quad W^3_{\mathbb{U}} = a_c\, \mathrm{Im}\, z^c, \qquad \text{quadratic:} \quad W^3_{\mathbb{U}} = \mathrm{Im}(A^a{}_0 z^a)$$

- Hidden symmetries

$$W^+_- = 2i\sqrt{2}\, \mathrm{e}^{\frac{k_\Omega}{2} - \phi}\, \xi^t \mathbb{C} Z, \qquad W^3_- = -W^3_{\mathbb{S}} - \mathrm{e}^{-2\phi}$$
$$W^+_{\hat\alpha} = \partial_\xi W^+_-, \qquad W^3_{\hat\alpha} = 2\, \partial_\xi W^3_-$$

Quaternionic isometries: prepotentials 1

Recall:

$$P^x = i_k\omega^x + W_k^x .$$

- Extra symmetries

$$i_+\omega^+ = 0, \qquad i_+\omega^3 = \frac{1}{2}\,\mathrm{e}^{\frac{\phi}{2}},$$

$$i_0\omega^+ = -\sqrt{2}\,\mathrm{e}^{\frac{K_\Omega}{2}+\phi}\,\xi^t\mathbb{C}Z, \qquad i_0\omega^3 = -\sigma\,\mathrm{e}^{\frac{\phi}{2}},$$

$$i_\alpha\omega^+ = -\sqrt{2}\,\mathrm{e}^{\frac{K_\Omega}{2}+\phi}\,\mathbb{C}Z, \qquad i_\alpha\omega^3 = -\frac{1}{2}\,\mathrm{e}^{\frac{\phi}{2}}\mathbb{C}\xi$$

- Duality symmetries

$$i_{\mathbb{U}}\omega^+ = \sqrt{2}\,\mathrm{e}^{\frac{K_\Omega}{2}+\phi}\,Z^t\mathbb{C}\mathbb{U}\xi,$$

$$i_{\mathbb{U}}\omega^3 = \frac{1}{4}\,\mathrm{e}^{2\phi}\,\xi^t\mathbb{C}\mathbb{U}\xi - \mathrm{e}^{K_\Omega}\,Z^t\mathbb{C}\mathbb{U}\bar{Z}$$

Quaternionic isometries: prepotentials 2

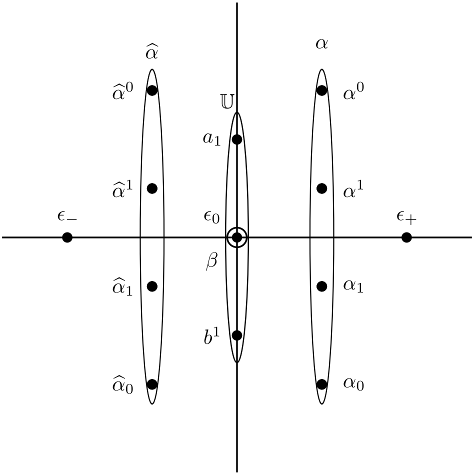
- Hidden symmetries

$$
\begin{aligned}
i_- \omega^+ &= \sqrt{2}\, \mathrm{e}^{\frac{K_\Omega}{2}+\phi} \big(\sigma\, Z^t \mathbb{C} \xi - 2i\, \mathrm{e}^{-2\phi}\, \xi^t \mathbb{C} Z - Z^t \mathbb{C} \partial_\xi W\big), \\
i_- \omega^3 &= \frac{1}{2}\, \mathrm{e}^{-2\phi} + \frac{1}{4}\, \mathrm{e}^{2\phi} \big(2\sigma^2 - 2W - \xi^t \mathbb{C} \partial_\xi W\big) - \mathrm{e}^{K_\Omega}\, Z^t \mathbb{C} \mathbb{S} \bar{Z},
\end{aligned}
$$

$$
\begin{aligned}
i_{\hat\alpha} \omega^+ &= -\sqrt{2}\, \mathrm{e}^{\frac{K_\Omega}{2}+\phi} (Z^t \mathbb{C} \xi)\, \mathbb{C} \xi - 2 \mathbb{C} \partial_\xi (i_- \omega^+), \\
i_{\hat\alpha} \omega^3 &= -\mathbb{C} \big(\sigma\, \mathrm{e}^{2\phi} \xi + \partial_\xi (i_{\hat\alpha} \omega^3)\big)
\end{aligned}
$$

Related work [1407.6956, Fré–Sorin–Trigiante]

Quaternionic isometries: Killing algebra



Outline: 5. BPS solutions

BPS equations

- BPS equations

$$\delta_Q \psi_\mu^\alpha = \delta_Q \lambda^{\alpha i} = \delta_Q \zeta^A = 0$$

- First order equations on bosonic fields
- Implies Einstein and scalar equations (but not Maxwell) [1005.3650, Hristov–Looyestijn–Vandoren]
- For static black holes, derived in [Halmagyi–Petrini–Zaffaroni, 1305.0730]

Gaugings

- Abelian gauging: $k_\Lambda^i = 0$
- Magnetic gaugings: introduce magnetic vector fields

$$\mathrm{d}\widetilde{A}_\Lambda = G_\Lambda, \qquad \mathrm{D} = \mathrm{d} + (\widetilde{A}_\Lambda \tilde{k}^{u\Lambda} - A^\Lambda k_\Lambda^u)\partial_u$$

  Complicated Lagrangian, but looks only at equations of motion [1012.3756, Dall'Agata–Gnecchi]
- Symplectic Killing vector

$$\mathcal{K} = \begin{pmatrix} \tilde{k}^\Lambda \\ k_\Lambda \end{pmatrix} = \mathcal{K}^u \partial_u = \Theta^{\mathcal{A}} k_{\mathcal{A}}$$

  $\Theta^{\mathcal{A}}$ gauging parameters (symplectic vectors) with constraints [hep-th/0507289, de Wit–Samtleben–Trigiante][0808.4076, Samtleben]
  $k_{\mathcal{A}} = \{h_\pm, h_0, h_\alpha, h_{\hat{\alpha}}, h_{\mathbb{U}}\}$
- Killing prepotentials

$$\mathcal{P}^x = \omega_u^x \mathcal{K}^u + \mathcal{W}^x$$

Ansatz

- Asymptotic AdS_4 (UV)

$$\mathrm{d}s^2 = -\frac{r^2}{R^2}\,\mathrm{d}t^2 + \frac{R^2}{r^2}\,\mathrm{d}r^2 + \frac{r^2}{R^2}\,\mathrm{d}\Sigma_g^2$$

- Near-horizon $\mathrm{AdS}_2 \times \Sigma_g$ (IR)

$$\mathrm{d}s^2 = -\frac{r^2}{R_1^2}\,\mathrm{d}t^2 + \frac{R_1^2}{r^2}\,\mathrm{d}r^2 + R_2^2\,\mathrm{d}\Sigma_g^2$$

- Σ_g Riemann surfaces of genus g
- Electric and magnetic charges

$$\mathcal{Q} = \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} = \frac{1}{4\pi}\int_{\Sigma_g} \begin{pmatrix} F^\Lambda \\ G_\Lambda \end{pmatrix}$$

- Define

$$\mathcal{Z} = \langle \mathcal{Q}, \mathcal{V} \rangle\,, \qquad \mathcal{L}^x = \langle \mathcal{P}^x, \mathcal{V} \rangle$$

AdS_4 BPS equations

Set $\mathcal{P}^1 = \mathcal{P}^2 = 0, \mathcal{P} \equiv \mathcal{P}^3$, then

$$\mathcal{P} = -2\,\mathrm{Im}\,(\bar{\mathcal{L}}\mathcal{V}), \qquad \mathcal{L} = \frac{i\,\mathrm{e}^{i\psi_0}}{R}, \qquad \langle \mathcal{K}^u, \mathcal{V}\rangle = 0$$

[1005.3650, Hristov–Looyestijn–Vandoren] [1312.2766, Halmagyi–Gnecchi]
Contract last with ω_u^x gives

$$\mathcal{L} - \langle \mathcal{W}, \mathcal{V}\rangle = 0.$$

$\mathcal{W} = 0 \rightarrow$ no regular solution.
Need non-trivial compensators from duality and hidden symmetries
$\rightarrow$ restriction on possible gaugings

Near-horizon BPS equations

- Equations solved for Fayet–Iliopoulos gauging ($n_h = 0$, $\mathcal{P}^x = \mathrm{cst}$) [1308.1439, Halmagyi] [1312.2766, Halmagyi–Gnecchi] [1408.2831, Halmagyi]
- $n_h \neq 0$: $\mathcal{P}^x = \mathcal{P}^x(q^u)$ which gives

$$\tau^i = \tau^i(p^\Lambda, q_\Lambda, \Theta^\mathcal{A}, q^u)$$

- Equations for q^u

$$\langle \mathcal{K}^u, \mathcal{V} \rangle = \langle \mathcal{K}^u, \mathcal{Q} \rangle = 0$$

  Solving them gives

$$\tau^i = \tau^i(p^\Lambda, q_\Lambda, \Theta^\mathcal{A}), \qquad q^u = q^u(p^\Lambda, q_\Lambda, \Theta^\mathcal{A})$$

- Entropy

$$S = \pi \sqrt{\mathcal{I}_4(\mathcal{P})}$$

Outline: 6. Conclusion

Achievements

- Symplectic expressions for hidden Killing vectors
- Compensators and prepotentials for all quaternionic isometries
- BPS equations with magnetic gaugings for full $N = 2$ matter-coupled supergravity
- Conditions for $N = 2$ AdS_4 vacua
- Framework to solve for AdS_4 and near-horizon geometries in a given model

Outlook

- Generalizes to homogeneous $\mathcal{M}_h$ and to non-abelian gaugings
- BPS equations for Taub–NUT black holes (FI gaugings)
- Add hypermultiplets to the general FI black hole solution [1408.2831, Halmagyi]
- BPS equations for near-horizon geometries of rotating black holes
- Study other vacua (Minkowski. . .)
- Generates charges using Demiański–Janis–Newman algorithm [1410.2602, H. E.] [1411.2030, H. E.–Heurtier] [1411.2909, H. E.] [1412.xxxx, H. E.–Heurtier]