NUT Black Holes in N = 2 Gauged Supergravity

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Collaborations with Nick Halmagyi (Lpthe) and Lucien Heurtier (Cpht, École Polytechnique)

Outline

Outline: 1. Introduction

Introduction

BPS equations

Demiański–Janis–Newman algorithm

Conclusion

Plebański–Demiański solution ('76)

Most general black hole solution [Plebański–Demiański '76]

- Einstein–Maxwell theory with cosmological constant Λ (equivalently pure $N = 2$ gauged supergravity)
- 6 parameters
  - mass m
  - NUT charge n
  - electric charge q
  - magnetic charge p
  - rotation a
  - acceleration α

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$$m + in, \qquad q + ip, \qquad a + i\alpha$$

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- BPS branches [hep-th/9203018, Romans] [hep-th/9512222, Kostelecky–Perry] [hep-th/9808097, Caldarelli–Klemm] [hep-th/0003071, Alonso-Alberca–Meessen–Ortín] [1303.3119, Klemm–Nozawa]

Motivations

Black holes

- sandbox for quantum gravity
- understand microstates from string theory
- adS/CFT correspondence

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NUT charge for AdS/CFT

- gauge dual: Chern–Simons on Lens spaces $S^3/\mathbb{Z}_n$ [1212.4618, Martelli–Passias–Sparks]
- fluid/gravity: NUT charge $\to$ vorticity [1206.4351, Caldarelli et al.]

Roadmap

Goals

- understand asymptotically adS_4 black holes
- Plebański–Demiański in $N = 2$ gauged supergravity with vector- and hypermultiplets

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- study simpler solution classes $\rightarrow$ BPS equations
- use a solution generating technique $\rightarrow$ Janis–Newman algorithm

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- study simpler solution classes $\rightarrow$ BPS equations
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This talk: focus on NUT charge (plus mass and dyonic), no hypermultiplet

Fields of $N = 2$ supergravity

- Gravity multiplet and n_v vector multiplets

$$\{g_{\mu\nu}, \psi_\mu^\alpha, A_\mu^0\}, \qquad \{A_\mu^i, \lambda^{\alpha i}, \tau^i\}, \qquad \begin{array}{c} \alpha = 1, 2 \\ i = 1, \ldots, n_v \end{array}$$

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- Lagrangian with Fayet–Iliopoulos gaugings

$$\mathcal{L}_{\text{bos}} = \frac{R}{2} + \frac{1}{4}\,\mathrm{Im}\,\mathcal{N}(\tau)_{\Lambda\Sigma}\,F^{\Lambda}_{\mu\nu}\,F^{\Sigma\mu\nu} - \frac{1}{8}\,\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}\,\mathrm{Re}\,\mathcal{N}(\tau)_{\Lambda\Sigma}\,F^{\Lambda}_{\mu\nu}\,F^{\Sigma}_{\rho\sigma} - g_{i\bar{\jmath}}(\tau)\,\partial_{\mu}\tau^i\partial^{\mu}\bar{\tau}^{\bar{\jmath}} - V(\tau)$$

Scalars: non-linear sigma model on special Kähler manifold (prepotential $F \to$ Kähler potential K)

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$$- g_{i\bar{\jmath}}(\tau)\, \partial_\mu \tau^i \partial^\mu \bar{\tau}^{\bar{\jmath}} - V(\tau)$$

Scalars: non-linear sigma model on special Kähler manifold (prepotential $F \to$ Kähler potential K)

- Electric and magnetic field strengths

$$F^\Lambda = \mathrm{d}A^\Lambda, \qquad \Lambda = 0, \ldots, n_v,$$
$$G_\Lambda = \star \left(\frac{\delta \mathcal{L}_{\rm bos}}{\delta F^\Lambda} \right) = \operatorname{Re} \mathcal{N}_{\Lambda\Sigma} F^\Lambda + \operatorname{Im} \mathcal{N}_{\Lambda\Sigma} \star F^\Lambda$$

Symplectic covariance

- Field strength and Maxwell equations

$$\mathcal{F} = \begin{pmatrix} F^\Lambda \\ G_\Lambda \end{pmatrix}, \qquad \mathrm{d}\mathcal{F} = 0$$

Maxwell equations invariant under $\mathrm{Sp}(2n_v + 2, \mathbb{R})$

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- Section

$$\mathcal{V} = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \qquad \tau^i = \frac{L^i}{L^0},$$

- Maxwell charges

$$\widehat{\mathcal{Q}} = \frac{1}{\mathrm{Vol}\,\Sigma} \int_\Sigma \mathcal{F} = \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}$$

- Fayet–Iliopoulos gaugings

$$\mathcal{G} = \begin{pmatrix} g^\Lambda \\ g_\Lambda \end{pmatrix}$$

  electric/magnetic charges of ψ_μ^α under $\mathrm{U}(1) \subset \mathrm{SU}(2)_R$

- covariant formalism for BPS equation [1012.3756, Dall'Agata–Gnecchi]

Quartic function

Symplectic vector A: order-4 homogeneous polynomial

$$I_4 = I_4(A, \tau^i)$$

Define symmetric 4-tensor

$$t_{MNPQ} = \frac{\partial^4 I_4(A)}{\partial A^M \partial A^N \partial A^P \partial A^Q}$$

Different arguments and gradient

$$\begin{aligned} I_4(A, B, C, D) &= t_{MNPQ} A^M B^N C^P D^Q \\ I_4'(A, B, C)^M &= \Omega^{MR} t_{RNPQ} A^N B^P C^Q \end{aligned}$$

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Quartic invariant

Symmetric space [hep-th/9210068, de Wit–Vanderseypen–Van Proeyen] [0902.3973, Cerchiai et al.]

$$\partial_i I_4(A) = 0$$

Quartic function: some identities

Arbitrary space

$$I_4(\mathcal{V}) = 0, \qquad I_4(\mathrm{Re}\,\mathcal{V}) = I_4(\mathrm{Im}\,\mathcal{V}) = \frac{1}{16}$$
$$\mathrm{Re}\,\mathcal{V} = 2\,I_4'(\mathrm{Im}\,\mathcal{V})$$

Symmetric space

$$I_4'(I_4'(A), A, A) = -8\,I_4(A)\;A$$
$$I_4'(I_4'(A), I_4'(A), A) = 8\,I_4(A)\;I_4'(A)$$
$$I_4'(I_4'(A)) = -16\,I_4(A)^2\;A$$

Quartic invariants

- cubic prepotential (symmetric very special Kähler manifold)

$$F = -D_{ijk}\,\tau^i\tau^j\tau^k$$

quartic invariant

$$I_4(\mathcal{Q}) = -\,(q_\Lambda p^\Lambda)^2 + \frac{1}{16}\,p^0\,\hat{D}^{ijk}q_iq_jq_k - 4\,q_0\,D_{ijk}p^ip^jp^k + \frac{9}{16}\,\hat{D}^{ijk}D_{k\ell m}q_iq_j\,p^\ell p^m$$

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- quadratic prepotential

$$F = \frac{i}{2}\,\eta_{\Lambda\Sigma}X^\Lambda X^\Sigma$$

quartic invariant

$$I_4(\mathcal{Q}) = \left(\frac{i}{2}\eta_{\Lambda\Sigma}p^\Lambda p^\Sigma + \frac{i}{2}\eta^{\Lambda\Sigma}q_\Lambda q_\Sigma\right)^2$$

Outline: 2. BPS equations

Ansatz

AdS–NUT dyonic black hole

$$\begin{aligned}
\mathrm{d}s^2 &= -\,\mathrm{e}^{2U}\big(\mathrm{d}t + 2n\,H(\theta)\,\mathrm{d}\phi\big)^2 + \mathrm{e}^{-2U}\mathrm{d}r^2 + \mathrm{e}^{2(V-U)}\,\mathrm{d}\Sigma_g^2 \\
A^\Lambda &= \tilde{q}^\Lambda(r)\big(\mathrm{d}t + 2n\,H(\theta)\,\mathrm{d}\phi\big) + \tilde{p}^\Lambda(r)\,H(\theta)\,\mathrm{d}\phi \\
\tau^i &= \tau^i(r)
\end{aligned}$$

$U = U(r)$, $V = V(r)$; Riemann surface Σ_g of genus g

$$\mathrm{d}\Sigma_g^2 = \mathrm{d}\theta^2 + H'(\theta)^2\,\mathrm{d}\phi^2, \qquad H(\theta) = \begin{cases} -\cos\theta & \kappa = 1 \\ \theta & \kappa = 0 \\ \cosh\theta & \kappa = -1 \end{cases}$$

with curvature $\kappa = \mathrm{sign}(1-g)$

NUT charge: preserves $\mathrm{SO}(3)$ isometry

Root structure

In general, e^{2V}: quartic polynomial

$$\mathrm{e}^{2V} = v_0 + v_1 r + v_2 r^2 + v_3 r^3 + v_4 r^4$$

- naked singularity: pair of complex conjugate roots, $v_3 = 0$
  $\rightarrow$ no horizon
- black hole: two real roots, $v_0 = 0$
  $\rightarrow$ horizon and finite temperature
- extremal black hole: real double root, $v_0 = v_1 = 0$
  $\rightarrow$ two coincident horizons, vanishing temperature, near-horizon $\mathrm{adS}_2 \times \Sigma_g$
- double extremal black hole: pair of real double roots, $v_0 = v_1 = 0$ and $v_3 = \sqrt{v_2 v_4}$
- ultracold black hole: real triple root, $v_0 = v_1 = v_2 = 0$
  [hep-th/9203018, Romans]

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Constant scalar black hole

Minimal gauged supergravity ($n_v = 0, \Lambda = -3g^2$)

$$\mathrm{e}^{2V} = g^2(r^2 + n^2)^2 + (\kappa + 4g^2n^2)(r^2 - n^2) - 2mr + P^2 + Q^2$$
$$\mathrm{e}^{2(V-U)} = r^2 + n^2, \qquad \tilde{q} = \frac{Qr - nP}{r^2 + n^2}, \qquad \tilde{p} = P$$

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1/4-BPS conditions [hep-th/0003071, Alonso-Alberca–Meessen–Ortín]

$$m = |2gnQ|, \qquad P = \pm \frac{\kappa + 4g^2n^2}{2g}$$

Two pairs of complex conjugate roots

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Two pairs of complex conjugate roots
Real root ⇒ extremal black hole

$$Q^2 = -2n^2(\kappa + 2g^2n^2)$$
$$r_1^- = r_2^- = \frac{\sqrt{1 - \kappa - 4g^2n^2}}{2\sqrt{2}g} > 0$$

BPS equations

Define

$$\widetilde{\mathcal{V}} = \mathrm{e}^{V-U}\,\mathrm{e}^{-i\psi}\,\mathcal{V}$$

1/4-BPS equations [1503.04686, H.E.–Halmagyi]

– differential (one vector, one scalar)

$$2\,\mathrm{e}^{V}\partial_r\,\mathrm{Im}\,\widetilde{\mathcal{V}} = -\mathcal{Q} + I_4'(\mathcal{G}, \mathrm{Im}\,\widetilde{\mathcal{V}}, \mathrm{Im}\,\widetilde{\mathcal{V}}) + 2n\kappa\mathcal{G}r$$

$$(\mathrm{e}^{V})' = -2\left\langle \mathrm{Im}\,\widetilde{\mathcal{V}}, \mathcal{G}\right\rangle$$

– algebraic (two scalars)

$$\mathrm{e}^{V}\left\langle \mathrm{Im}\,\widetilde{\mathcal{V}}, \partial_r\,\mathrm{Im}\,\widetilde{\mathcal{V}}\right\rangle = 2\left\langle \mathrm{Im}\,\widetilde{\mathcal{V}}, \mathcal{Q}\right\rangle - 3n\kappa\,\mathrm{e}^{V} + 4n\kappa r\left\langle \mathcal{G}, \mathrm{Im}\,\widetilde{\mathcal{V}}\right\rangle$$

$$\langle \mathcal{Q}, \mathcal{G}\rangle = \kappa \in \mathbb{Z}$$

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Note: BPS selects ± 1 for Dirac condition

- dynamical variables: only V and $\mathrm{Im}\,\widetilde{\mathcal{V}}$ appear [1405.4901, Katmadas]
- $\mathcal{Q}$: integration constants from Maxwell equations
- valid even for non-symmetric manifold (static case: see [1509.00474, Katmadas–Tomasiello])

Solution

Ansatz

$$\mathrm{e}^{2V} = v_0 + v_1 r + v_2 r^2 + v_3 r^3 + v_4 r^4$$
$$\mathrm{Im}\,\widetilde{\mathcal{V}} = \mathrm{e}^{-V}\big(A_0 + A_1 r + A_2 r^2 + A_3 r^3\big)$$

V based on constant scalar solution and [Plebański–Demiański '76]

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V based on constant scalar solution and [Plebański–Demiański '76]

Generic features

$$v_{p+1} = \frac{1}{p+1}\,\langle \mathcal{G}, A_p\rangle\,, \qquad p \geq 0$$
$$A_p = a_{p1}\,\mathcal{G} + a_{p2}\,\mathcal{Q} + a_{p3}\,I_4'(\mathcal{G}) + a_{p4}\,I_4'(\mathcal{G},\mathcal{G},\mathcal{Q}) + a_{p5}\,I_4'(\mathcal{G},\mathcal{Q},\mathcal{Q}) + a_{p6}\,I_4'(\mathcal{Q})$$

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Given $(\mathcal{G}, \mathcal{Q})$ and one constraint: analytic solution for symmetric space [1503.04686, H.E.–Halmagyi]

$$a_{pi} = a_{pi}(\mathcal{G}, \mathcal{Q}, n)$$

In particular (from adS_4 asymptotics)

$$A_3 = \frac{1}{4}\,\frac{I_4'(\mathcal{G})}{\sqrt{I_4(\mathcal{G})}}, \qquad v_4 = \frac{1}{R_{\mathrm{adS}}^2} = \sqrt{I_4(\mathcal{G})}$$

Extremal solution

- $a_{p1} = a_{p2} = 0$

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- near-horizon constraint [1308.1439, Halmagyi]

$$0 = 4\, I_4(\mathcal{P}) I_4(\mathcal{P}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})^2 + 4\, I_4(\mathcal{Q}) I_4(\mathcal{Q}, \mathcal{P}, \mathcal{P}, \mathcal{P})^2 \\ - I_4(\mathcal{P}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q}) I_4(\mathcal{P}, \mathcal{P}, \mathcal{Q}, \mathcal{Q}) I_4(\mathcal{Q}, \mathcal{P}, \mathcal{P}, \mathcal{P})$$

plus Dirac condition $\rightarrow 2n_v$ independent charges

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  plus Dirac condition $\rightarrow$ $2n_v$ independent charges
- entropy

$$S = \pi \sqrt{I_4(\mathrm{Im}\, \widetilde{\mathcal{V}})}\Big|_{r=r_h} = \pi R^2_{\Sigma_g}$$

  and

$$R^4_{\Sigma_g} = \frac{I_4(\mathcal{Q}, \mathcal{Q}, \mathcal{G}, \mathcal{G}) \pm \sqrt{I_4(\mathcal{Q}, \mathcal{Q}, \mathcal{G}, \mathcal{G})^2 - I_4(\mathcal{Q}) I_4(\mathcal{G})}}{I_4(\mathcal{G})}$$

Independent roots and constant scalars

STU model with constant scalars and

$$P^0 = Q_i = P, \qquad Q_0 = -P^i = Q$$

Section

$$A_0 = \frac{n\kappa(P-1)}{2g}\,\mathcal{G} + \frac{n\kappa}{8g^3}\,I_4'(\mathcal{G}),$$
$$A_1 = \frac{Q}{2g}\,\mathcal{G} + \frac{P-3gn^2}{8g^3}\,I_4'(\mathcal{G}),$$
$$A_2 = \frac{n\kappa}{2}\,\mathcal{G}, \qquad A_3 = \frac{I_4'(\mathcal{G})}{4\sqrt{I_4(\mathcal{G})}},$$

Metric

$$\mathrm{e}^{2V} = 2\Big(P^2+Q^2+g^2n^4-2gn^2P+4gn\kappa Qr+2(3gn^2-gP)r^2+gr^4\Big)$$

Spinor phase

$$\sin\psi = \mathrm{e}^{U-2V}\big(gr^3+(-P+3gn^2)r+n\kappa Q\big)$$

Outline: 3. Demiański–Janis–Newman algorithm

Introduction

Demiański–Janis–Newman algorithm [Newman–Janis '65] [Demiański–Newman '66] [Demiański '72]

- idea: **complex** change of coordinates $\to$ new charges (rotation, NUT)
- off-shell (derived metric is **not** necessarily solution)
- two prescriptions: Newman–Penrose formalism (more rigorous), direct complexification (quicker) [Giampieri '90] [1410.2602, H.E.]

Introduction

Demiański–Janis–Newman algorithm [Newman–Janis '65] [Demiański–Newman '66] [Demiański '72]

- idea: **complex** change of coordinates $\rightarrow$ new charges (rotation, NUT)
- off-shell (derived metric is **not** necessarily solution)
- two prescriptions: Newman–Penrose formalism (more rigorous), direct complexification (quicker) [Giampieri '90] [1410.2602, H.E.]
- main achievement: discovery of Kerr–Newman solution [Newman et al. '65]
- before 2014: defined only for the metric, 3 examples fully known without fluid (and 2 partly)
  (Kerr, BTZ, singly-rotating Myers-Perry)

Needs for supergravity

- gauge fields
- complex scalar fields
- topological horizons
- dyonic charges
- NUT charge: understand the complexification

Needs for supergravity

- ✓ gauge fields [1410.2602, H.E.]
- ✓ complex scalar fields [1501.02188, H.E.–Heurtier]
- ✓ topological horizons [1411.2909, H.E.]
- ✓ dyonic charges [1501.02188, H.E.–Heurtier]
- ✓ NUT charge: understand the complexification [1411.2909, H.E.]
- ✓ *bonus:* higher dimensions [1411.2030, H.E.–Heurtier]

Janis–Newman algorithm

Giampieri's prescription

$$1)\quad \mathrm{d}t = \mathrm{d}u - k(r)\,\mathrm{d}r \quad \Longrightarrow \quad g_{rr} = 0$$

$$2)\quad u, r \in \mathbb{C}, \qquad f_i(r) \to \tilde{f}_i = \tilde{f}_i(r, \bar{r}) \in \mathbb{R}$$

$$3)\quad u = u' + i\,G(\theta), \qquad r = r' - i\,F(\theta)$$

$$4)\quad i\,\mathrm{d}\theta = \sin\theta\,\mathrm{d}\phi$$

$$5)\quad \left(\begin{cases} \mathrm{d}t' = \mathrm{d}u' - g(r)\mathrm{d}r \\ \mathrm{d}\phi' = \mathrm{d}\phi' - h(r)\mathrm{d}r \end{cases} \Longrightarrow \quad \begin{cases} g_{tr} = 0 \\ g_{r\phi} = 0 \end{cases} \right)$$

Janis–Newman algorithm

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\begin{aligned}
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\end{aligned}
$$

Complexification rules for $f \to \tilde{f}$

$$
\begin{aligned}
r &\longrightarrow \frac{1}{2}\,(r + \bar{r}) = \mathrm{Re}\, r \\
\frac{1}{r} &\longrightarrow \frac{1}{2}\left(\frac{1}{r} + \frac{1}{\bar{r}}\right) = \frac{\mathrm{Re}\, r}{|r|^2} \\
r^2 &\longrightarrow |r|^2
\end{aligned}
$$

Simple example (metric only)

Reissner–Nordström

$$\begin{aligned} \mathrm{d}s^2 &= -f\,\mathrm{d}t^2 + f^{-1}\,\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \\ &= -f\,\mathrm{d}u^2 - 2\,\mathrm{d}u\mathrm{d}r + r^2\mathrm{d}\Omega^2, \end{aligned} \qquad f = 1 - \frac{2m}{r} + \frac{q^2}{r^2}$$

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$$u = u' + i\,a\cos\theta, \qquad r = r' - i\,a\cos\theta$$

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Kerr–Newman

$$
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\end{aligned}
$$

$$
\tilde{f} = 1 - \frac{2mr'}{\rho^2} + \frac{q^2}{\rho^2}, \qquad \rho^2 \equiv |r|^2 = r'^2 + a^2\cos^2\theta
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Gauge fields

Reissner–Nordström

$$A = \frac{q}{r}\,\mathrm{d}t = \frac{q}{r}\left(\mathrm{d}u - f^{-1}\mathrm{d}r\right)$$

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$$A_r = 0$$

$\rightarrow$ missing step in [Newman et al. 65']!
(other approach: [1407.4478, Keane])

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Kerr–Newman

$$A = \frac{qr'}{\rho^2}\,(\mathrm{d}u' - a\sin^2\theta\,\mathrm{d}\phi')$$

Scalar fields

Example: axion–dilaton pair

$$\tau = \mathrm{e}^{-2\phi} + i\sigma$$

Static

$$\mathrm{e}^{2\phi} = 1 + \frac{R}{r}, \qquad \sigma = 0$$

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Static

$$\mathrm{e}^{2\phi} = 1 + \frac{R}{r}, \qquad \sigma = 0$$

Need to transform the complex field as a single entity

$$\tilde{\tau} = 1 + \frac{R}{r' - i\,a\cos\theta} = 1 + \frac{R\,(r' + i\,a\cos\theta)}{r'^2 + a^2\cos^2\theta}$$

Generates axion

$$\mathrm{e}^{2\phi} = 1 + \frac{R\,r'}{\rho^2}, \qquad \sigma = \frac{R\,a\cos\theta}{\rho^2}$$

Complex parameters

- presence of magnetic charge: use the central charge

$$Z = q + ip$$

example: dyonic Reissner–Nordström

$$A = \text{Re}\,\frac{Z}{r}\,\mathrm{d}t + \text{Im}\,Z\cos\theta\,\mathrm{d}\phi$$

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$$Z = q + ip$$

example: dyonic Reissner–Nordström

$$A = \mathrm{Re}\,\frac{Z}{r}\,\mathrm{d}t + \mathrm{Im}\,Z\cos\theta\,\mathrm{d}\phi$$

- adding a NUT charge: complexify the mass, shift horizon curvature

$$m = m' + i\kappa n, \qquad \kappa = \kappa' - \frac{4\Lambda}{3}\,n^2$$

Full DJNA

Ansatz: Einstein–Maxwell plus scalar fields [1411.2909, H.E.]

$$\mathrm{d}s^2 = -f_t(r)\,\mathrm{d}t^2 + f_r(r)\,\mathrm{d}r^2 + f_\Omega(r)\,(\mathrm{d}\theta^2 + H'(\theta)^2\,\mathrm{d}\phi^2)$$
$$A = f_A(r)\,\mathrm{d}t, \qquad \chi = \chi(r)$$

DJN transformation

$$r = r' + i\,F(\theta), \qquad u = u' + i\,G(\theta), \qquad i\,\mathrm{d}\theta = H'(\theta)\,\mathrm{d}\phi$$

Resulting metric

$$\mathrm{d}s^2 = -\tilde{f}_t\big(\mathrm{d}t + \omega H\,\mathrm{d}\phi\big)^2 + \frac{\tilde{f}_\Omega}{\Delta}\,\mathrm{d}r^2 + \tilde{f}_\Omega\big(\mathrm{d}\theta^2 + \sigma^2 H^2 \mathrm{d}\phi^2\big)$$
$$\Delta = \frac{\tilde{f}_\Omega}{\tilde{f}_r}\,\sigma^2, \qquad \omega = G' + \sqrt{\frac{\tilde{f}_r}{\tilde{f}_t}}\,F', \qquad \sigma^2 = 1 + \frac{\tilde{f}_r}{\tilde{f}_\Omega}\,F'^2$$
$$A = \tilde{f}_A\left(\mathrm{d}t - \frac{\tilde{f}_\Omega}{\sqrt{\tilde{f}_t\tilde{f}_r}\,\Delta}\,\mathrm{d}r + G'H\,\mathrm{d}\phi\right), \qquad \tilde{\chi} = \tilde{\chi}(r,\theta)$$

Solutions for F and G

Strategy: find F and G by solving the equations of motion in one example, declare this transformation always valid [Demiański '72]

– $\Lambda = 0$

$$F(\theta) = n - a\,H(\theta) + c\left(1 + H(\theta)\,\ln\frac{H'(\theta/2)}{H(\theta/2)}\right)$$

$$G(\theta) = -2\kappa\,n\,\ln H'(\theta) + \kappa\,a\,H(\theta) - \kappa\,c\,H(\theta)\,\ln\frac{H'(\theta/2)}{H(\theta/2)}$$

– $\Lambda \neq 0$

$$F(\theta) = n, \qquad G(\theta) = -2\kappa\,n\,\ln H'(\theta)$$

Note: the solution is *not* unique

Parameters: n: NUT charge, a: rotation, c: ?

New examples

- Kerr–Newman–NUT
- dyonic Kerr–Newman
- Yang–Mills Kerr–Newman [Perry '77]
- adS–NUT Schwarzschild
- BPS solutions from $N = 2$ ungauged supergravity [hep-th/9705169, Behrndt–Lüst–Sabra]
- non-extremal rotating black hole in T^3 model [hep-th/9204046, Sen] [gr-qc/9907092, Yazadjiev]
- SWIP solutions [hep-th/9605059, Bergshoeff–Kallosh–Ortín]
- charged Taub–NUT–BBMB with Λ [1311.1192, Bardoux–Caldarelli–Charmousis]
- 5d Myers–Perry [Myers–Perry '86]
- BMPV [hep-th/9602065, Breckenridge–Myers–Peet–Vafa]

Outline: 4. Conclusion

AdS–NUT black holes

Demiański–Janis–Newman algorithm:

- (almost) all examples can be embedded in $N = 2$ supergravity
- non-extremal adS–NUT black hole in gauged $N = 2$ sugra with $F = -i\,X^0 X^1$ [Klemm–Rabbiosi, private communication]
- consequence of supersymmetry / U-duality / string theory?
- derive 1/4-BPS black holes with $n \neq 0$ from the ones with $n = 0$ in [1408.2831, Halmagyi]?

Achievements

- general analytic solution of 1/4-BPS dyonic adS–NUT black holes with running scalars in $N = 2$ FI supergravity
- extend DJN algorithm to all fields with spin ≤ 2 and topological horizons
- define DJNA with m, n, p, q, a (a only for $\Lambda = 0$)

Outlook

- understand properties of 1/4-BPS adS(–NUT) black holes
- compute free energy and compare with localization
- 1/2-BPS adS–NUT black holes
- BPS solutions with rotation and acceleration
- Demiański–Janis–Newman algorithm
  - more $N = 2$ gauged supergravity solutions
  - $d \geq 6$ Myers–Perry
  - charged $d > 4$ black holes (in Einstein–Maxwell)
  - multicenter solutions
  - black rings
  - (fake) superpotentials in supergravity

Thank you!