Journal club: Chaos and black holes

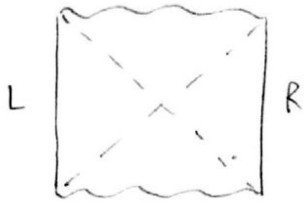
Program:

1. AdS/CFT and AdS eternal BH
2. Scrambling and butterfly effect (shocks)
3. Lyapunov exponent (OTOC)

Reviews: Harlow 1409.1231

1. AdS/CFT

a) AdS eternal BH



$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$= -f(r)\,du\,dv + r^2 d\Omega^2 \qquad \text{(Eddington-Finkelstein)}$$

$du = dt - f(r)^{-1} dr \; ; \; dv = dt + f(r)^{-1} dr$

Define tortoise coord: $dr_* = \dfrac{dr}{f(r)} \implies \begin{cases} u = t - r_* \\ v = t + r_* \end{cases}$

Horizon at $r = r_s$: $f(r_s) = 0$

- surface gravity: $K = \dfrac{f'(r_s)}{2}$ — entropy: $S = \dfrac{A}{4l_p^2} = \dfrac{A}{4G\hbar}$
- Hawking temperature: $\beta = \dfrac{2\pi}{\hbar K}$ — area: $A = \underset{Vol(S^{d-1})}{4\pi} r_s^2$

Concrete: $f(r) = 1 - \dfrac{2GM}{r} + \dfrac{r^2}{l^2}$ $\qquad l$: adS scale

for $l = \infty$, $r_s = 2GM$, $K = \dfrac{1}{4GM}$, $\beta = \dfrac{8\pi GM}{\hbar}$, $S = \dfrac{4\pi G M^2}{\hbar}$

Kruskal-Szekeres coord: smooth at horizon, maximal coord ext. $\qquad \frac{U}{V}, UV$

$$U = T - X = -e^{-Ku} \; ; \; V = T + X = e^{Kv}$$

$$ds^2 = \frac{f(r)}{K^2}\frac{dU\,dV}{UV} + r^2 d\Omega^2 \qquad \text{note: } U = -e^{-K(t-r_*)}, \; V = e^{K(t+r_*)}$$

$$= -\frac{4f(r)}{f'(r_s)^2} e^{-f'(r_s) r_*} \underbrace{dU\,dV}_{(dT^2 - dX^2)} + r^2 d\Omega^2$$

$V = Re^{w}$
$U = -Re^{-w}$

Horizon: $UV = 0 \quad (r_* \to -\infty) \; ; \qquad UV = -1$: sing. or bdy

b) Holography

$AdS_{d+1} \times M \longleftrightarrow CFT_d$
$\uparrow$
compact

geometry $\longleftrightarrow$ state

ex: - vacuum ($\Rightarrow M=0$, max. sym. space) $\to$ adS
$\longleftrightarrow$ vacuum of CFT : $|0\rangle$
- BH ($M \neq 0$, $\Rightarrow T \neq 0$)
$\longleftrightarrow$ thermal state in CFT $|0\rangle_\beta$

- acting on state of CFT with op.
$\longleftrightarrow$ deformation of the geometry

AdS eternal BH : dual to two decoupled copies $CFT_L \times CFT_R$
(Maldacena, hep-th/0106112)
$\to$ thermofield double

$$|\psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\frac{\beta E_n}{2}} |n\rangle_L \otimes |n\rangle_R$$

$|n\rangle_{L,R}$: eigenstates of $CFT_{L,R}$

2. Scrambling

Def: Scrambling time t_* = time for a perturbation to be so mixed in the system that one needs to look at a fraction $O(1)$ of the states to reconstruct it.

Conjecture (Susskind - Sekino, 0808.2096)

$$t_* \gtrsim \hbar \beta \ln S$$

1101.6048

Fast scramblers reach bound: $t_* \sim \hbar \beta \ln S$
(equiv to $d = \infty$ system)

ex: - BH, Rindler space, dS
- matrix brane models, quantum circuits

Standard system: expects $t_* = t_D$ (diffusion time)
scrambling done through diffusion

Estimation: $\sqrt{\beta t_D} \sim l \sim N^{1/d}$; entropy extensive $S \sim N$
$\hookrightarrow$ system size ($\sim$ hypercubic lattice)

$$\Rightarrow t_D \sim c \, \beta \, S^{\frac{2}{d}}$$

- class. gas/liquid: $c = l_0 \bar{p}$ (size)
- strongly corr. quant. fluid: $c \sim \hbar$

Note: for a BH $\hbar \beta \ln S \sim \frac{2\pi}{K} \ln \frac{\pi r_s^2}{G\hbar} \sim \frac{1}{K} \ln \frac{1}{K}$

$$\overset{Sch}{\sim} r_s \ln r_s \qquad (G = 1)$$

Harlow: for solar mass BH, $t_* \sim 10^{-4} s$

First argument for BH: stretched horizon ($\rightarrow$ membrane paradigm)

$$r_h = r_s + l_P$$

(see SS '08} Hayden-Preskill 0708.4025)

Second: AdS/CFT.

The appearance of the log comes from the relation between the proper time $\tau \sim T$ and Sch time t (= blueshift):

$$\Delta\tau \sim \frac{1}{K} e^{K\Delta t} \quad (\text{set } X = cst,\ UV|_{r_s} = 0,\ f(r_s) = 0) \qquad \text{not clear}$$

hence $\Delta\tau \sim 1$ (Planck time) $\Rightarrow$ $\Delta t \sim r_s \ln r_s$

On the geometry a highly energetic particle is like a shock wave: $T_{uu} \sim \delta(u)$

(Shenker-Stanford: 1306.0622, 1312.3296, 1409.8180

Injected at time t_w a quantum of energy E:

$\rightarrow$ effect is $\frac{E}{M} e^{\frac{2\pi t_w}{\beta}}$ (blueshift of E)

For min energy $E = T$, this is $O(1)$ for

$$t_w = t_*$$

Shock wave: spherically sym. null matter with asymp. energy $E \ll M$ at time t_w

$\rightarrow$ split spacetime in 2: (U,V) and $(\hat{U},\hat{V})$

Give shift $\hat{V} = V + \alpha$, then $\alpha \simeq 1 \Rightarrow t = t_*$

Butterfly effect: localize in S^{D-2}: $T_{uu} \sim \delta(u)\, \hat{\delta}^{D-2}(\Omega)$

growth (spatial) of disturbance

Shock wave ansatz:

$$ds^2 = \ell^2 \left[- A(UV)\, dU dV + B(UV)\, dx^i dx^i + A(UV)\, \delta(U)\, h(x)\, dU^2 \right]$$

$$\tilde{V} = V + h \quad ; \qquad T_{uu} \sim e^{\frac{2\pi t_w}{\beta}} \delta(U)\, a_0(x)$$

$a_0(x)$: support in $x \lesssim 1$

$\int a_0\, dx \sim 1$

Einstein eq : $\left(- \Delta^{(D-2)} + \mu^2 \right) h(x) \sim e^{\frac{2\pi t_w}{\beta}} a_0(x)$

$$\mu^2 = \frac{(D-1)(D-2)}{2}$$

for $|x| \gg 1$; $h(x) \sim \dfrac{e^{\frac{2\pi}{\beta}(t_w - t_*) - \mu |x|}}{|x|^{\frac{d-2}{2}}}$.

Butterfly velocity: $v_B = \dfrac{2\pi}{\beta \mu} = \sqrt{\dfrac{D-1}{2(D-2)}}$

note: entanglement speed

$$v_E = \frac{\sqrt{d}\,(d-2)^{\frac{1}{2} - \frac{1}{d}}}{\left(2(d-1)\right)^{1-\frac{1}{d}}}$$

Dual CFT: perturbation of $|\psi\rangle$: $W(t_w)|\psi\rangle$

- Evaluation of the effect done by evaluating mutual information

$$I(A,B) = S(A) + S(B) - S(A\cup B) \qquad \text{measures corr.}$$

$\rightarrow$ compute S using Ryu-Takayanagi prescription

$$S_A = \min_{\Sigma:\partial\Sigma = A} \frac{\cancel{\mathrm{Vol}}(\Sigma)}{4G}$$

In shock wave background find $I = 0$ for $t_w \simeq t_*$

- For localized perturb. look at

$$C(t_w, |x-y|) = \langle\psi| [W_x(t_w), W_y(0)]^\dagger [W_x(t_w), W_y(0)] |\psi\rangle$$
$$= 2 - 2\,\mathrm{Re}\,\langle\psi_w|\psi_w'\rangle$$

3. Lyapunov

Interest in : $C(t) = -\langle [W(t), V(0)]^2 \rangle$

Square : avoids cancellation between phases

Motivation : take $W = x$, $V = p$ ↱ Lyapunov exp.

$$C(t) = -\langle [x(t), p(0)]^2 \rangle \sim \hbar^2 \{x(t), p(0)\}_{PB} \sim \hbar^2 e^{2\lambda_L t}$$

since for a class. chaotic system

$$\{x(t), p(0)\} = \frac{\partial x(t)}{\partial x(0)} \sim e^{\lambda t}$$

↳ sensitivity of $x(t)$ to ini cond.

Expand : find OTO

refs : 1612.01278 (QM), 1603.03020 (CFT), 1703.09435

Conj. bound : $\lambda_L \leq \frac{2\pi}{\beta\hbar} = K$

note : $C(t_*) \sim 1 \Rightarrow t_* \sim \frac{1}{\lambda_L} \ln \hbar$

$\rightarrow \lambda_L = \frac{2\pi}{\beta}$

One would find :

$$\langle W(x,t) V(0,0) W(x,t) V(0,0) \rangle \sim 1 - \frac{1}{N} e^{\lambda_L t - \lambda_L \frac{x}{v_B}}$$

$$\tau_L = \frac{1}{\lambda_L}$$

SYK model : diffusion cst $D \leq v_B^2 \tau_L$