Effective string field theory

Outline
1. Motivations
2. Scalar field theory
3. Classical SFT
4. Effective SFT: perturbative approach
   . Effective SFT: coalgebra approach
5. Heterotic EFT and localization

Presentation
- already online talk, happy to come in person
- also working on 2d gravity, SYK/tensor models, black holes and supergravity
- applications of ML
    - lattice systems, in particular QFT/QCD (Chernodub)
    - integrability (Samuel Belliard)
    - string theory and algebraic topology (student in Turin)

Papers: 1912.05463, 2006.16270
with: Carlo Maccaferri, Martin Schnabl, Jakub Vošmera

Motivations

Standard model of particle physics / general relativity
- fundamental quanta = point particles
- simplest approach: classical/quantum mechanics
  (worldline / 1st quantized)
  = describe particle embedded in spacetime

- modern language: field theory (2nd quantized)
  = describe particles as oscillations of field
  filling all spacetime

String theory: same evolution
- start with worldsheet description
  could go far thanks to properties of 2d theories (symmetries, CFT, Riemann surfaces)
- problems lead to find a field theory

Limitations of worldsheet description:
- on-shell
  * divergences associated with on-shell internal particles
    solution: renormalization ⟵ off-shell
    (mass renormalization, vacuum shift)
  * consistency properties (unitarity, crossing sym., analyticity ...)
- no potential to minimize for finding vacua
  need consistent backgrounds to compactify from $D=10$ to $D=4$
- perturbative
  ⟶ non-perturbative, collective, thermal effects?
- background dependence

Main research areas:
1. construction and structure of the action
   → formal and general properties
   → how to make explicit computations?
2. consistency of string theory
3. improvement of worldsheet computations
4. classical solutions of open SFT
   note: Witten's SFT has very peculiar properties making this possible, but not useful to understand SFT in general

Review: 1703.06410, book to appear (Springer)

Goals: study effective SFT
- focus on low-energy effective action ("massless" modes)
  → ignore ∞ number of "massive" fields
  a priori not useful for pheno.
- understand general structure (symmetries, observables)
- deform action with source
- compute higher-derivative corrections

Related work:
- Koyama - Okawa - Suzuki: 2006.16449, 2006.16710
- Maccaferri - Merlano: 1801.07607, 1905.04958

Scalar field theory

String field theory may look esoteric (not using standard QFT language)

$\longrightarrow$ consider toy model for making contact

N real scalar fields φ_i (momentum space, Euclidean):

$$S = \frac{1}{2} \sum_{i=1}^{N} \int \frac{d^D k}{(2\pi)^D} \varphi_i(k) (k^2 + m_i^2) \varphi_i(-k)$$
$$+ \frac{1}{3!} \int \prod_{i=1}^{3} \frac{d^D k_i}{(2\pi)^D} V^{(3)}_{i_1 i_2 i_3}(k_1, k_2, k_3) \varphi_{i_1}(k_1) \varphi_{i_2}(k_2) \varphi_{i_3}(k_3)$$
$$+ \frac{1}{4!} \int \prod_{i=1}^{4} \frac{d^D k_i}{(2\pi)^D} \underbrace{V^{(4)}_{i_1 \cdots i_4}}_{\text{vertices in Feynman rules}}(k_1, \dots, k_4) \varphi_{i_1}(k_1) \cdots \varphi_{i_4}(k_4)$$

[Weinberg, vol. 2, sec. 21.6]

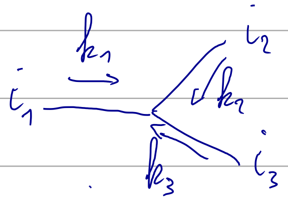
vertices in Feynman rules

φ^4 theory with $N = 1$:

$$V^{(3)} = 0 \quad , \qquad V^{(4)}(k_1, \dots, k_4) = \lambda\, \delta^{(D)}(k_1 + \dots + k_4)$$

Propagator: $\Delta_{ij}(k) = \dfrac{\delta_{ij}}{k^2 + m_i^2}$

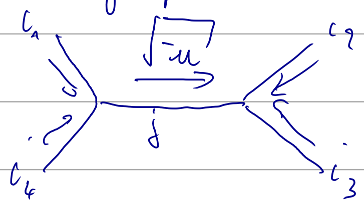
Amplitudes

$$A^{(3)}_{i_1 i_2 i_3}(k_1, k_2, k_3) = $$ 

$$= -V^{(3)}_{i_1 i_2 i_3}(k_1, k_2, k_3)$$

$$A^{(4)}_{i_1 \cdots i_4}(k_1, \ldots, k_4) = -V^{(4)}_{i_1 \cdots i_4}(k_1, \ldots, k_4) + \mathcal{S}^{(4)}_{i_1 \cdots i_4}(k_1, \ldots, k_4)$$

where $\mathcal{S}^{(4)}$ contains graphs with propagators (s-, t-, u-channels)

$$\mathcal{S}^{(4)}_{i_1 \cdots i_4} = $$ 

$$+ \text{perms}$$

$$= \sum_j V_{i_1 i_4 j}(k_1, k_4, \sqrt{-u}) \frac{1}{-u + m_j^2} V_{i_2 i_3 j}(k_2, k_3, \sqrt{-u})$$

$$+ \text{perms}$$

Imagine you get $A^{(3)}, A^{(4)} \ldots$ from some method
(scattering amplitude program, worldsheet path integral, ..)
$\longrightarrow$ reverse engineering of vertices if propagator known
$\hookrightarrow$ action

Ket representation

Introduce complete basis first-quantized states

$$\mathcal{H} = \text{Span}\{|\phi_i(k)\rangle\}$$

eigenstates of momentum operator p, mass operator m

$$p|\phi_i(k)\rangle = k|\phi_i(k)\rangle \quad , \quad m|\phi_i(k)\rangle = m_i|\phi_i(k)\rangle$$

Define an inner product:

$$\langle\phi_i(k)|\phi_j(k')\rangle = (2\pi)^D \delta_{ij}\, \delta^{(D)}(k+k')$$

Worldline position representation: operator X conj to p

$$|\phi_i(k)\rangle = \varepsilon_i\, e^{ik\cdot X}|0\rangle \longrightarrow \text{vertex operators}$$

On-shell states: 0-eigenvalue of worldline Hamiltonian

$$H|\phi_i(k)\rangle = (k^2+m_i^2)|\phi_i(k)\rangle = 0$$

$\rightarrow$ field theory eq. of motion

Field ket = linear combination of 1st quantized states

$$|\varphi\rangle = \int \frac{d^Dk}{(2\pi)^D}\, \varphi_i(k)\, |\phi_i(k)\rangle$$

Rewrite action:

- kinetic term given by worldline $H: \mathcal{H} \to \mathcal{H}$
- interactions: define operators $\mathcal{V}_n : \mathcal{H}^{\otimes n} \to \mathbb{C}$

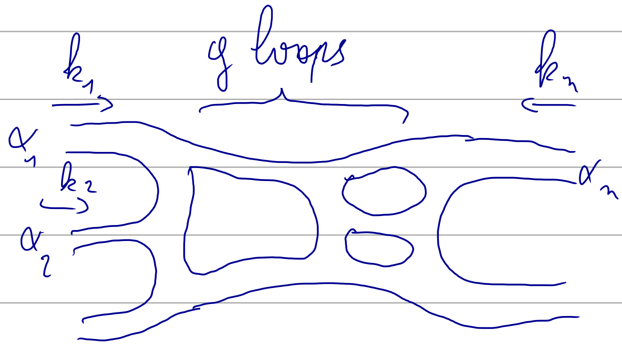
$$\mathcal{V}_n(|\phi_{i_1}(k_1)\rangle, \dots, |\phi_{i_n}(k_n)\rangle) = V^{(n)}_{i_1,\dots,i_n}(k_1,\dots,k_n)$$

$$\Rightarrow S = \frac{1}{2}\underbrace{\langle\varphi|H|\varphi\rangle}_{\mathcal{V}_2(\varphi^2)} + \frac{1}{3!}\mathcal{V}_3(\varphi^3) + \frac{1}{4!}\mathcal{V}_4(\varphi^4)$$

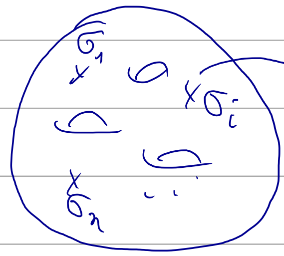
Classical string field theory

Worldsheet path integral

$$A_{g,n}(k_1,\dots,k_n)_{\alpha_1,\dots,\alpha_n} = \sum_{\text{surfaces}}$$



$$= \sum_{\text{surfaces}}$$



vertex op. $V_{\alpha_i}(k_i;\sigma_i)$

genus-g Riemann surface with n punctures

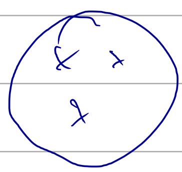
$$= \int \frac{dg_{ab}\, dX}{\text{Vol gauge}} e^{-S_m[g,X]} \left\langle \prod_{i=1}^{n} \int d^2\sigma_i \sqrt{g}\, V_{\alpha_i}(k_i;\sigma_i) \right\rangle$$

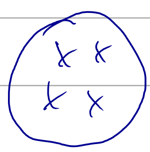
$$\overset{\text{gauge fixing}}{=} \int_{\mathcal{M}_{g,n}} d^{M_{g,n}} t \left\langle \text{ghosts} \prod_{i=1}^{n} V_{\alpha_i}(k_i) \right\rangle_{\Sigma_{g,n}(t)}$$

↳ 2d CFT correlations functions

↳ moduli space
= set of Riemann surfaces which are topologically inequivalent

Moduli parameters $t_\lambda \in \mathcal{M}_{g,n} \simeq \{$ size of holes, puncture positions $\}$

$\Sigma_{0,3} =$  $\mathcal{M}_{0,3} = 1$

$\Sigma_{0,4} =$  $\mathcal{M}_{0,4} = \mathbb{C}$

Feynman rules and action

On-shell condition: $L_0 |\Phi\rangle = 0$

$S_{free} = \frac{1}{2} \langle \Phi, L_0 \Phi \rangle$ $\qquad$ Φ: string field (no ghosts)

↳ worldsheet Hamiltonian

↳ inner product built from CFT

Propagator: $\qquad$ $L_0 = \alpha' k^2 + \hat{L}_0$

↳ $\alpha' m^2$

$$\frac{1}{L_0} = \int_0^\infty ds\, e^{-sL_0}$$

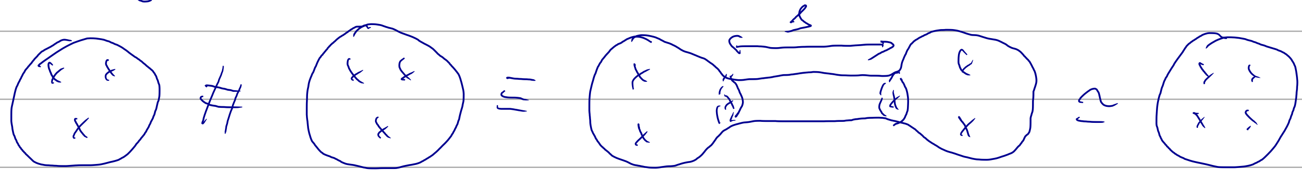
↳ Schwinger parameter

L_0 = generator of dilatation

$\longrightarrow e^{-sL_0}$ inserts a piece of worldsheet of length s

Mathematical operation $\#$ = plumbing fixture

But: some $\Sigma_{0,4}$ can be described as two $\Sigma_{0,3}$ connected by a tube:



note: twist angle θ

vary s and θ $\Longrightarrow$ $\mathcal{M}_{0,3} \# \mathcal{M}_{0,3} \equiv \mathcal{F}_{0,4} \subset \mathcal{M}_{0,4}$

missing region $\mathcal{V}_{0,4}$ ≡ fundamental vertex region

$$\tilde{\mathcal{F}}_{g,n}(k_1,\dots,k_n)_{\alpha_1\cdots\alpha_n} \equiv \int_{\tilde{\mathcal{F}}_{g,n}} d^{M_{g,n}}t \left\langle \text{ghosts} \times \prod_{i=1}^{n} V_{\alpha_i}(k_i) \right\rangle$$

↳ all surfaces $\Sigma_{g,n}$ obtained by gluing lower-order surfaces

interpreted as sum of Feynman diagrams with propagators

$$\mathcal{V}_{g,n}(k_1,\dots,k_n)_{\alpha_1\cdots\alpha_n} \equiv \int_{\mathcal{V}_{g,n}} d^{M_{g,n}}t \left\langle \text{ghosts} \times \prod_{i=1}^{n} V_{\alpha_i}(k_i) \right\rangle$$

↳ remaining surfaces $\mathcal{V}_{g,n} \equiv \mathcal{M}_{g,n} - \tilde{\mathcal{F}}_{g,n}$

interpreted as fundamental vertices

String field action:

$$S = \sum_{g,n\geq 0} \frac{\hbar^{2g} g_s^{2g+n-2}}{n!} \mathcal{V}_{g,n}(\Phi^n)$$

$$= \sum_{g,n\geq 0} \frac{\hbar^{2g} g_s^{2g+n-2}}{n!} \langle \Phi, \ell_{g,n-1}(\Phi^{n-1}) \rangle$$

↳ string products

In practice: very hard to find all $\mathcal{V}_{g,n}$

note: precise def. of corr. function involve local coordinates on $\Sigma_{g,n}$, which are $\neq$ for $\mathcal{V}_{g,n}$ and $\tilde{\mathcal{F}}_{g,n}$
also very hard

Effective SFT

string field $\Psi \in \mathcal{H}$ (include ghosts)
↳ Hilbert space of string states

Classical SFT action:

$$S = \frac{1}{2}\langle \Psi, Q\Psi \rangle + \langle \Psi, V(\Psi) \rangle$$

$$V(\Psi) = \sum_{n\geq 2} \frac{1}{(n+1)!} \ell_n(\Psi^n)$$ (omit index 0 : $\ell_{0,n} \to \ell_n$)

Eq. of motion:

$$\mathcal{E}(\Psi) \equiv Q\Psi + V'(\Psi) = 0$$

$$V(\Psi) \equiv \int_0^1 dt\, V'(t\Psi)$$

String products satisfy L_∞ relations: $\ell_1 = Q$

$$0 = \sum_{n_1+n_2=n} \ell_{n_1+1}(A_1, \dots, A_k, \ell_{n_2}(A_{k+1}, \dots, A_n))$$

First relations:

$Q^2 = 0$ nilpotent

$$Q\,\ell_2(A_1, A_2) \pm \ell_2(QA_1, A_2) \pm \ell_2(A_1, QA_2) = 0$$

Q derivative of ℓ_2

$$Q\,\ell_3(A_1, A_2, A_3) \pm \ell_3(QA_1, A_2, A_3) \pm \dots$$

$$= \pm\, \ell_2(\ell_2(A_1, A_2), A_3) \pm \dots$$

failure of Q to be a deriv. of ℓ_3 = failure of Jacobi identity

$\Longrightarrow$ gauge invariance of S:

$$\delta_\Lambda \Psi = Q\Lambda + \sum_{n\geq 2} \frac{1}{n!} \ell_n(\Psi^n, \Lambda)$$

Split $\mathcal{H}$ using projector P:

$$[L_0, P] = [Q, P] = [b_0, P] = 0$$

$$P^+ = P \quad , \quad \ker \hat{L}_0 \subset \mathrm{Im}\, P \qquad \bar{P} \equiv 1 - P$$

- light states $P\mathcal{H}$
- heavy states $\bar{P}\mathcal{H}$

example: projector on massless states (open string)

$$\hat{P}_0 = e^{-\infty \hat{L}_0} \qquad \text{zero-momentum: } P_0 = e^{-\infty L_0}$$

Procedure to get effective action:

1. Siegel gauge fixing of heavy fields
2. integrate out heavy fields
3. check out-of-Siegel gauge constraints
4. integrate out light auxiliary fields

Integrate out heavy fields

gauge fixing: needed to invert propagator

Siegel gauge projector:

$$\Pi_s = b_0 c_0 \qquad \bar{\Pi}_s = c_0 b_0$$

Field decomposition:

$$\Psi = \varphi + R_\downarrow + R_\uparrow$$

$$\varphi = P\Psi \qquad R_\downarrow = \bar{P}\Pi_s\Psi \qquad R_\uparrow = \bar{P}\bar{\Pi}_s\Psi$$

BRST charge decomposition:

$$Q = c_0 L_0 - b_0 M_+ + \hat{Q}$$

Eq. of motion:

$$\mathcal{E}_\varphi(\Psi) \equiv P\,\mathcal{E}(\Psi) = Q\varphi + PV'(\Psi)$$

$$\mathcal{E}_{R_\downarrow}(\Psi) \equiv \bar{P}\bar{\Pi}_s\,\mathcal{E}(\Psi) = c_0 L_0 R_\downarrow + \cancel{\hat{Q}R_\uparrow} + \bar{P}\bar{\Pi}_s V'(\Psi)$$

$$\mathcal{E}_{R_\uparrow}(\Psi) \equiv \bar{P}\Pi_s\,\mathcal{E}(\Psi) = \hat{Q}R_\downarrow \cancel{- b_0 M_+ R_\downarrow} + \bar{P}\Pi_s V'(\Psi)$$

Siegel gauge for heavy field:

$$b_0(\bar{P}\Psi) = 0 \implies R_\uparrow = 0$$

Propagator: $\Delta = \dfrac{b_0}{L_0}$ $\qquad \{Q, \Delta\} = \bar{P}_0$

$$\mathcal{E}_{R_\downarrow} = 0 \implies R_\downarrow = -\frac{b_0}{L_0}\bar{P}V'(\varphi + R_\downarrow)$$

$\longrightarrow$ solve for $R_\downarrow = R_\downarrow(\varphi)$

L_0 still singular in $\bar{P}\mathcal{H}$

$\longrightarrow$ effective theory, momentum cut-off $\alpha' k^2 \ll \min \hat{L}_0$

$$\frac{1}{L_0} = \frac{1}{\alpha' k^2 + \hat{L}_0} = \frac{1}{\hat{L}_0} \sum_{n\geq 0} (-1)^n \alpha'^n k^{2n} \frac{1}{\hat{L}_0^n}$$

α' expansion from massive fields $\qquad \hookrightarrow$ higher-deriv.

Effective action:

$$S_{eff} = \frac{1}{2} \langle \varphi, Q\varphi \rangle + \langle \varphi, PV(\Psi_{eff}) \rangle$$
$$+ \left\langle \bar{\Pi}_s V'(\Psi_{eff}), \frac{b_0}{L_0} \bar{P} \left(\frac{V'(\Psi_{eff})}{2} - V(\Psi_{eff}) \right) \right\rangle$$

where $\Psi_{eff} = \varphi + R_\downarrow(\varphi)$

But: cannot recover from it $R_\uparrow$ eom = out-of-Siegel eq.

$$\mathcal{G}_{R_\uparrow} = \hat{Q} R_\downarrow + \bar{P}\Pi_s V'(\Psi_{eff}) = 0$$

$\longrightarrow$ impose beside action as constraint

$\sim$ Gauss constraints

Result: automatic if eom of φ holds

$$\mathcal{G}_{eff,\varphi} = 0 \implies \mathcal{G}_{R_\uparrow} = 0$$

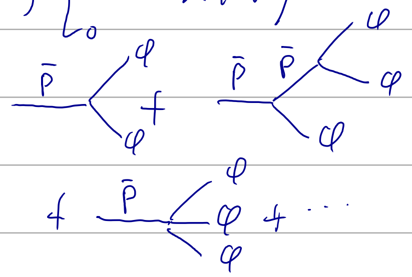
Perturbative solution

Expand fields and potentials ($\mu \ll 1$ pert. parameter):

$$\varphi = \sum_{n\geq 1} \mu^n \varphi_n, \qquad R_\varphi = \sum_{n\geq 1} \mu^n R_n, \qquad V' = \sum_{n\geq 2} \mu^n V'_n$$

Solve order by order in μ and resum:

$$R_\varphi(\varphi) = -\frac{1}{2}\frac{b_0}{L_0}\bar{P} l_2(\varphi^2) + \frac{1}{2}\frac{b_0}{L_0}\bar{P} l_2\left(\varphi, \frac{b_0}{L_0}\bar{P} l_2(\varphi^2)\right) - \frac{1}{3!}\frac{b_0}{L_0}\bar{P} l_3(\varphi^3) + O(\varphi^4) =$$



$$\varphi = \mu\varphi_1 + \mu^2\varphi_2 + O(\mu^3)$$

Effective action:

$$S_{\text{eff}} = \frac{1}{2}\langle \varphi, Q\varphi\rangle + \frac{1}{3!}\langle \varphi, P l_2(\varphi^2)\rangle + \frac{1}{4!}\langle \varphi, P l_3(\varphi^3)\rangle - \frac{1}{8}\left\langle \Pi_s l_2(\varphi^2), \frac{b_0}{L_0}\bar{P} l_2(\varphi^2)\right\rangle + O(\varphi^5)$$

Eq. of motion:

$$\mathcal{E}_{\text{eff},\varphi} = Q\varphi + \sum_{n\geq 2}\frac{1}{n!}\hat{l}_n(\varphi^n)$$

effective L_∞ structure:

$$\hat{l}_1 A = PQA \qquad \hat{l}_2(A_1, A_2) = P l_2(A_1, A_2)$$

$$\hat{l}_3(A_1, A_2, A_3) = P l_3(A_1, A_2, A_3) \pm P l_2\left(A_1, \frac{b_0}{L_0}\bar{P} l_2(A_2, A_3)\right) \pm \text{perms}$$

$\rightarrow$ eff. gauge invariance $\qquad \bar{P}\lambda = 0$

$$\delta_\lambda \varphi = Q\lambda + \hat{l}_2(\varphi, \lambda) + \frac{1}{2}\hat{l}_3(\varphi^2, \lambda) + O(\varphi^3)$$

Integrate out light auxiliary fields

Focus on open string, $\bar{P} = \hat{P}_0$ (keep massless fields)

States with $\hat{L}_0 = 0$ at $N_{gh} = 1$:

$$\Psi_A \equiv \frac{\sqrt{2}}{\alpha'} A_\mu(k)\, c\, i\partial X^\mu e^{ikX} \qquad \Psi_B = \frac{B(k)}{\sqrt{2}}\, \partial c\, e^{ikX}$$

$\hookrightarrow$ gauge field — primary if $k \cdot A = 0$, on-shell if $k^2 = 0$

$\hookrightarrow$ Nakanishi–Lautrup aux. field, not primary

Free action:

$$S = \frac{1}{2}\int \frac{d^D k}{(2\pi)^D}\left[A_\mu(k)\, k^2 A^\mu(-k) - B(k)B(-k) + 2k \cdot A(k) B(-k)\right]$$

Siegel gauge condition + constraint:

$$\begin{cases} b_0 \Psi_B = 0 \\ \hat{Q}\Psi_A = 0 \end{cases} \Longrightarrow \begin{cases} B(k) = 0 \\ k \cdot A = 0 \end{cases}$$

but loose gauge inv.

To preserve gauge invariance, integrate out NL field.

$$\Pi_\Delta \mathcal{E}_{eff}(\Psi) = 0 \Longrightarrow \Psi_B = c_0 M_- \left(\hat{Q}\Psi_A - PV'(\Psi_A + \Psi_B)\right)$$

$\hookrightarrow$ algebraic propagator

$SU(1,1)$ algebra: $[M_+, M_-] = \hat{N}_{gh}$, $[\hat{N}_{gh}, M_\pm] = \pm 2 M_\pm$

$\hookrightarrow$ ghost # without zero-mode

solve $\Psi_B = \Psi_B(\Psi_A) = O(\Psi_A)$

Better approach: field redefinition to make state with A_μ primary

$$\hat{\varphi}_A = \frac{A_\mu(k)}{\sqrt{2}}\left(\frac{2}{\alpha'} c\, i\partial X + k^\mu \partial c\right) e^{ikX}$$

$$\varphi_B = \frac{\beta(k)}{\sqrt{2}} \partial c \, e^{ikX} \qquad \beta \equiv B - k\cdot A$$

Can be implemented with a new projector $\Pi = \Pi_s + \dots$

Free action:

$$S = \frac{1}{2}\int \frac{d^D k}{(2\pi)^D}\left[A_\mu(k)\left(k^2 \eta^{\mu\nu} - k^\mu k^\nu\right) A_\nu(-k) - \beta(k)\,\beta(-k)\right]$$

Integrate out: $\varphi_\beta = O(\varphi_A^2)$

Coalgebra description
- much more powerful
- give expression at all orders
- uniformize the different steps

Heterotic SFF and localization

Use WZW form of the action (no L_∞ structure)

$$S_{eff}(\varphi) = \frac{1}{2}\langle \eta_0\varphi, Q\varphi\rangle + \frac{1}{3!}\langle \eta_0\varphi, l_2(\varphi, Q\varphi)\rangle$$
$$+ S^{(4)}_{eff}(\varphi) + O(\varphi^5)$$

$$S^{(4)}_{eff}(\varphi) = \frac{1}{4!}\langle \eta_0\varphi, l_2(\varphi, l_2(\varphi, Q\varphi))\rangle + \frac{1}{4!}\langle \eta_0\varphi, l_3(\varphi, Q\varphi, Q\varphi)\rangle$$
$$- \frac{1}{8}\left\langle l_2(\eta_0\varphi, Q\varphi), \xi_0 \frac{b_0^+}{L_0^+}\bar{P}\, l_2(\eta_0\varphi, Q\varphi)\right\rangle$$

η, ξ : ghosts from bosonization of super-ghosts (β, γ)

l_n : bosonic products

Assume:

- zero momentum
- $\bar{P} = P_0$ (massless states)
- global $N=2$ supersymmetry

$N=1$ primary $=$ sum of charged short $N=2$ primaries

$\varphi = \varphi^+ + \varphi^-$ $\quad$ ↳ R-sym.

Conservation of R-charge and ghost #:

$$S_{eff} = S^{(4)}_{eff} = -\frac{1}{8}\langle l_2(\varphi^-, \eta_0\varphi^-), P_0 l_2(\varphi^+, Q\varphi^+)\rangle + (+ \Leftrightarrow -)$$
$$- \frac{1}{8}\langle l_2(\varphi^-, \varphi^+), P_0 l_2(\eta_0\varphi^-, Q\varphi^+)\rangle + (+ \Leftrightarrow -)$$
$$+ O(\varphi^5)$$

$P_0 \sim e^{-\infty L_0^+}$ $\quad$ infinitely long projector

Terms with projectors of finite length cancel with contact terms
$\longrightarrow$ no need to know l_3

Moreover, form of l_2 is irrelevant because P_0 selects leading term from OPE:

$$P_0 l_2(A_1, A_2) = b_0^- \delta(L_0^-) \{A_1 A_2\}_{0,0}(0,0)|0\rangle$$

$\hookrightarrow$ coeff of $z^0 \bar{z}^0$

Reduce computation to 2-point functions.

General primary (except ghost dilaton):

$$\varphi = c\bar{c}\, V_{\frac{1}{2},1}\, \xi e^{-\varphi}$$

$\hookrightarrow$ $(\frac{1}{2}, 1)$ primary

Define:

$$\mathbb{H}^{\pm}_{1,1}(z,\bar{z}) = \lim_{\varepsilon,\bar{\varepsilon}\to 0} 2\bar{\varepsilon}\; V^{\pm}_{\frac{1}{2},1}(z+\varepsilon, \bar{z}\mp\bar{\varepsilon})\, V^{\pm}_{\frac{1}{2},1}(z-\varepsilon, \bar{z}-\bar{\varepsilon})$$

$$\mathbb{H}_{0,1}(z,\bar{z}) = \lim_{\varepsilon,\bar{\varepsilon}\to 0} |2\varepsilon|^2\; V^{+}_{\frac{1}{2},1}(z+\varepsilon, \bar{z}\mp\bar{\varepsilon})\, V^{-}_{\frac{1}{2},1}(z-\varepsilon, \bar{z}-\bar{\varepsilon})$$

$$\Longrightarrow S_{\text{eff}} = \frac{1}{4}\langle \mathbb{H}^{+}_{1,1}, \mathbb{H}^{-}_{1,1}\rangle + \frac{1}{4}\langle \mathbb{H}_{0,1}, \mathbb{H}_{0,1}\rangle + O(\varphi^5)$$

eom: $\quad \mathbb{H}^{\pm}_{1,1} = \mathbb{H}_{0,1} = 0 \qquad$ ADHM constraints

Flat background: $\mathbb{H}^{\pm}_{1,1}$, $\mathbb{H}_{0,1}$ contains only the gauge field, not the metric or B-field (consistent since deriv. coupling)

$$S^{(4)}_{\text{eff}} = -\frac{1}{16C}\, \text{tr}\, [A_\mu, A_\nu]^2$$

$\hookrightarrow$ Dynkin index

Conclusion

Achievements:

- understand better out-of-Siegel gauge constraints
- " " the role of auxiliary fields
- structure of effective SFT at all orders
  (bosonic/superstring, open/closed)
- develop localization for explicit computations

Outlook:

- compute interesting quartic eff. actions with α' corrections
- generalize to open-closed SFT
- compute ghost-dilaton contributions