Momentum integration in string field theory

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Review + lecture notes: De Lacroix, Kashyap, Verma

Introduction

* World-sheet theory

g-loop N-point amplitude

$$A_N = $$  $$= \int \prod_{i=1}^{6g-6+2N} dm_i \; F(m)$$

m_i: parameters of moduli space $\mathcal{M}_{g,N}$ of the genus g Riemann surface with N punctures $\Sigma_{g,N}$

F: correlation function

$$F = \Big\langle \prod_{a=1}^{N} \underbrace{V_a}_{\text{integrated vertex operator}} \times \underbrace{\text{ghosts} \times \overbrace{\text{PCO}}^{\text{superconformal ghosts}}}_{\text{independent of } \{V_a\}} \Big\rangle_{\Sigma_{g,N}}$$

$m_i \sim$ Schwinger parameters s_i

Recall in field theory: $\dfrac{1}{k^2+m^2} = \displaystyle\int_0^\infty ds\; e^{-s(k^2+m^2)}$

* Three types of divergences for A_N:
  - IR: regions where $m_i \longrightarrow \infty$ ($k^2+m^2=0$ or $k^2+m^2<0$)
  - UV: " " $m_i \longrightarrow 0$ (can be seen after integration over k)
  - spurious: " " m_i finite but A_N still diverges
    (due to superconformal ghosts, absent in QFT and bosonic string)
    except sugra

* Problems with worldsheet formulation

- divergences for large m:
  - artificial for $k^2 + m^2 < 0$
  - genuine for $k^2 + m^2 = 0$

    IR divergence: quantum effects shift vacuum and masses
- on-shell (BRST/conformal invariance): prevents renor

  → prevents using tools from QFT relying on going off-shell (renormalization, proofs of unitarity, crossing symmetry...)
- one graph at each loop level, yields only real result but expects imaginary amplitudes (unitarity, unstable resonances...)
- computations only for protected states (BPS, symmetry...)

There is ad hoc/off-shell prescriptions, but this is not fully satisfactory

→ build a string field theory

* String field theory (SFT)

Regular QFT with infinite # of fields s.t.

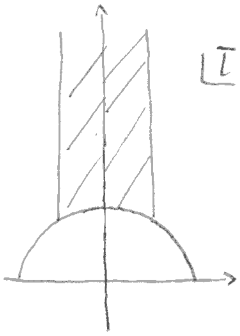
- correlation functions/amplitudes agree with the ones of the worldsheet formulation when the latter can be made finite
- genuine (IR) divergences agree but can be handled with usual QFT tools

Each Feynman graph = integral over part of $\mathcal{M}_{g,N}$

Notes:

- construction by reverse engineering: on-shell → off-shell → graphs → SFT
- no intrinsic/constructive formulation (i.e. 1) abstract/formal, 2) no minimal area form)

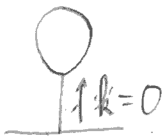
More on divergences



* UV divergences: absent from string theory because $\mathcal{M}_{g,n}$ does not contain the region $m_i \to 0$

* spurious poles: superconformal ghost CFT is non-unitary
  physically: breakdown of gauge fixing
  $\to$ can be avoided using

* IR divergence: in string theory = degeneration limit (long tubes)
  - tadpole for massless particle: vacuum shift
  - mass renormalization (resummation of 1PI graphs)



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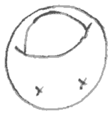
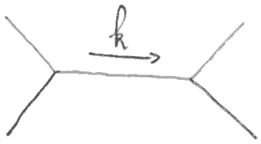


$m=0 \implies \left.\frac{1}{k^2}\right|_{k=0} = \infty$



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momentum conservation

$\implies \left.\frac{1}{k^2+m^2}\right|_{k^2=-m^2} = \infty$

Field theory and correlation functions

* *Action*: $\{\varphi_A\}$ are the physical fields, D non-compact dimensions

$$S = \sum_A \int d^D k \; \varphi_A(-k)\, K_A(k)\, \varphi_A(k) + \sum_{n=0}^{\infty} \sum_{\{A_1,\dots,A_n\}} \int d^D k_1 \cdots d^D k_n \; \delta^{(D)}(k_1 + \dots + k_n) \times$$

$$\times V_n^{\{A\}}(k_1,\dots,k_n)\, \varphi_{A_1}(k_1) \cdots \varphi_{A_n}(k_n)$$

forget dependence on A_i

Propagator: $K(k)^{-1} = \dfrac{P_1(k)}{k^2+m^2}$ $\longrightarrow$ polynomial

Vertices: $V_n(k_1,\dots,k_n) = \int dy \; e^{-\sum_{i,j} g_{ij}(y)\, k_i \cdot k_j} \; P(k_1,\dots,k_n; y)$

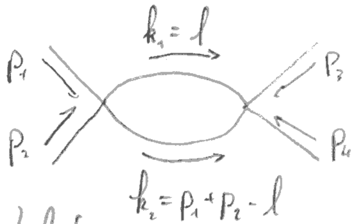
y: moduli ; g_{ij} is positive definite

* *Adding stubs* (rescaling of local coordinates).

ignore $\longrightarrow$ multiply vertex by $\exp\left(-\sum_i \Lambda_i(y)(k_i^2 + m_i^2)\right)$

- if $g_{ij}(y)$ was not positive definite then tune Λ_i
- $e^{-\sum_i \Lambda_i m_i^2}$ damps the contribution from the infinite # of fields (grows as e^{cm})

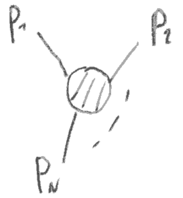
* *Correlation functions*



Momenta: $\{p_\alpha\}$ external, $\{k_i\}$ internal, $\{\ell_s\}$ loops

$\hookrightarrow$ linear combinations of $\{p_\alpha\}$ and $\{\ell_s\}$

label part. by its p_i

$G_N(p_1,\dots,p_N) =$  $\sim \int \prod_s d^D \ell_s \; e^{-\sum_{r,s} G_{rs}\, \ell_r \ell_s} \prod_i \dfrac{1}{k_i^2 + m_i^2}$

G_{rs} is positive definite $\longrightarrow$ the integral:

- over spatial momenta $\vec{\ell}_s$ converge
- over energies E_s diverge

This prevents using usual QFT tools and proofs directly (Wick rotation, $i\varepsilon$ prescription, analyticity, unitarity, crossing symmetry ...)

Momentum integration prescription

- multiply external energies by $u \in \mathbb{C}$: $(E_\alpha, \vec{p}_\alpha) \longrightarrow (uE_\alpha, \vec{p}_\alpha)$
- define the correlation for $u \in i\mathbb{R}$, $l_s^0 \in i\mathbb{R}$
  $\longrightarrow$ every propagator has $k_i^2 + m_i^2 \in \mathbb{C}$ and there is no poles
- take the limit $u \longrightarrow 1$ and deform the contour to keep the poles on the same side (keep the end points at $\pm i\infty$)

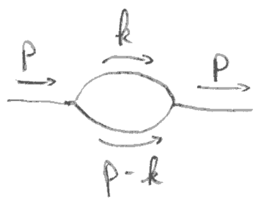
One can show that this is well defined (correlation function is analytic) if

$$\mathrm{Re}\, u \geq 0 , \quad \mathrm{Im}\, u > 0$$

Note: it is equivalent with the $i\varepsilon$ prescription from Berera ('94) and Witten ('13)

$$\frac{1}{k^2+m^2} \longrightarrow \int_0^{i\infty} dt \; e^{-t(k^2+m^2-i\varepsilon)}$$

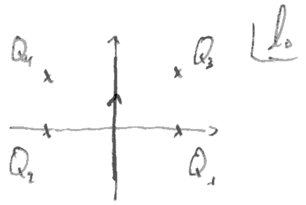
* Example



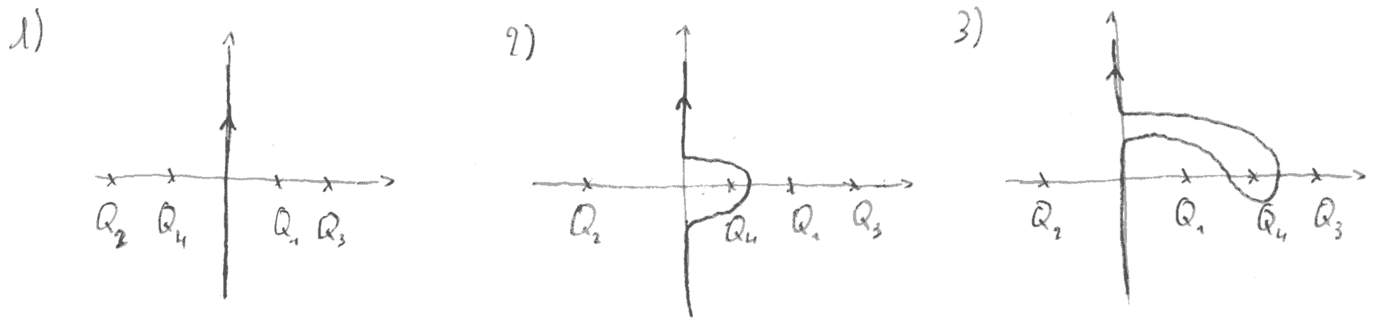
$$G \sim \frac{1}{k^2+m^2} \, \frac{1}{(p-k)^2+m^2}$$

4 poles Q_i

- $u \in i\mathbb{R}$



- $u \longrightarrow 1$: three cases



Unitarity

S-matrix: $S = 1 - iT$

Unitarity $S^\dagger S = 1 \implies i(T - T^\dagger) = T^\dagger T = \sum_n T^\dagger |n\rangle\langle n| T \qquad (1)$

$\{|n\rangle\}$: complete set of physical asymptotic states

Method: 't Hooft, Veltman ('93)
- Cutkosky rules: get RHS with unphysical states
- Ward identity: decoupling of unphysical states

Note: Cutkosky rule:
- take a graph and cut through loop propagators, replace it with $\delta(k^2+m^2)\theta(k^0)$
- amplitude = sum over all possible ways of cutting