PhD defense
Black holes in $N = 2$ supergravity

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Papers

- H. Erbin and N. Halmagyi. “Abelian Hypermultiplet Gaugings and BPS Vacua in $N = 2$ Supergravity”. *JHEP* 2015.5 (May 2015), 1409.6310.
- H. Erbin and N. Halmagyi. “Quarter-BPS Black Holes in AdS_4-NUT from $N = 2$ Gauged Supergravity”. Accepted in *JHEP* (Mar. 2015), 1503.04686.
- H. Erbin. “Janis-Newman algorithm: simplifications and gauge field transformation”. *General Relativity and Gravitation* 47.3 (Mar. 2015), 1410.2602.
- H. Erbin and L. Heurtier. “Five-dimensional Janis-Newman algorithm”. *Classical and Quantum Gravity* 32.16 (July 2015), p. 165004, 1411.2030.
- H. Erbin. “Deciphering and generalizing Demiański-Janis-Newman algorithm”. Submitted to *Classical and Quantum Gravity* (Nov. 2014), 1411.2909
- H. Erbin and L. Heurtier. “Supergravity, complex parameters and the Janis-Newman algorithm”. *Classical and Quantum Gravity* 32.16 (July 2015), p. 165005, 1501.02188.

Outline

Outline: 1. Introduction

Modèle standard et relativité générale

Modèle standard :

- interactions entre particules élémentaires
- trois forces (électromagnétisme, faible, forte)
- théorie quantique

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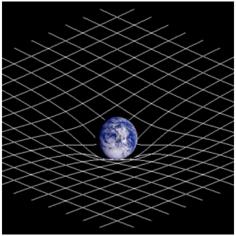
- force gravitationnelle = déformation de l'espace-temps
- nécessaire si vitesse/gravité élevées
- théorie classique



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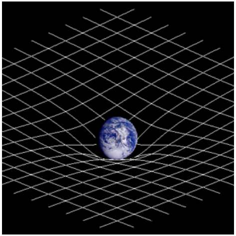

Objectifs de la physique moderne :

- quantifier la gravité
- décrire ensemble le modèle standard et la gravité

$\rightarrow$ théorie des cordes

Supersymétrie

Deux types de particules :

- les bosons : transmettent les forces (e.g. le photon)
- les fermions : constituent la matière (e.g. l'électron)

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$$Q_{\text{susy}} \left|\text{boson}\right\rangle = \left|\text{fermion}\right\rangle , \qquad Q_{\text{susy}} \left|\text{fermion}\right\rangle = \left|\text{boson}\right\rangle$$

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Supergravité

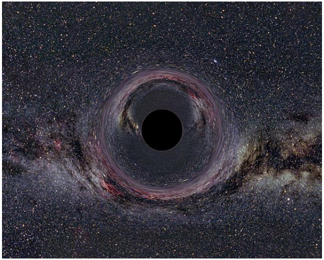
$$\text{relativité générale} + \text{supersymétrie}$$

- limite de la théorie des cordes
- unification interactions/gravité
- meilleur comportement quantique

N : nombre de Q_{susy} différents
Choix : $N = 2$ (compromis liberté/simplicité)

Trous noirs

- champ gravitationnel extrême
- horizon : limite au-delà de laquelle rien ne peut s'échapper
- centre = singularité (gravité infinie)
- description complète : nécessite une gravité quantique



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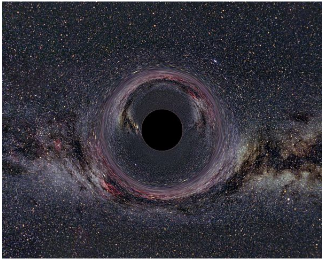

- bac à sable pour tester les théories de gravité quantique
- peu de paramètres : ressemble à une particule

Outline: 2. Motivations

Plebański–Demiański solution ('76)

Most general black hole solution [Plebański–Demiański '76]

- Einstein–Maxwell theory with cosmological constant Λ
- equivalently pure $N = 2$ gauged supergravity
- 6 parameters
  - mass m
  - NUT charge n
  - electric charge q
  - magnetic charge p
  - rotation j
  - acceleration a
- natural pairing as complex parameters

$$m + in, \qquad q + ip, \qquad j + ia$$

Motivations

(AdS) black holes

- sandbox for quantum gravity
- understand microstates from string theory
- adS/CFT correspondence

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Black hole: interpolation

$$\text{magnetic } \mathrm{adS} \text{ (UV)} \to \text{near-horizon geometry (IR)}$$

AdS_4 and near-horizon geometry $\to$ supergravity solutions

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magnetic adS (UV) $\rightarrow$ near-horizon geometry (IR)

AdS_4 and near-horizon geometry $\rightarrow$ supergravity solutions

Roadmap

Goals

- understand asymptotically adS_4 black holes
- Plebański–Demiański in $N = 2$ gauged supergravity with vector- and hypermultiplets

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- understand asymptotically adS_4 black holes
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Two strategies

- study simpler solution classes $\rightarrow$ BPS equations
- use a solution generating technique $\rightarrow$ Janis–Newman algorithm

BPS equations

- BPS equations

$$\text{fermions} = 0, \qquad \delta_Q(\text{fermions}) = 0$$

- background preserves part of supersymmetry
- first order differential equations on bosonic fields
- imply (most of) the equations of motion
  $N = 2$: give Einstein and scalar equations, but not Maxwell
  [1005.3650, Hristov–Looyestijn–Vandoren]

Outline: 3. Supergravity and BPS solutions

Introduction

Motivations

Supergravity and BPS solutions

Demiański–Janis–Newman algorithm

Conclusion

$N = 2$ supergravity

Algebra

$$\left\{Q_\alpha, \bar{Q}^\beta\right\} \sim \delta_\alpha{}^\beta P, \qquad [J, Q_\alpha] \sim \gamma \cdot Q_\alpha,$$
$$\{Q_\alpha, Q_\beta\} \sim \varepsilon_{\alpha\beta} Z, \qquad [R, Q_\alpha] \sim U_\alpha{}^\beta Q_\beta$$

P momentum, Z central charge, J angular momentum
automorphism U, R-symmetry $\mathrm{U}(2)_R$

$N = 2$ supergravity

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Field content

- gravity multiplet

$$\{g_{\mu\nu}, \psi_\mu^\alpha, A_\mu^0\}, \qquad \alpha = 1, 2$$

- n_v vector multiplets

$$\{A_\mu^i, \lambda^{\alpha i}, \tau^i\}, \qquad i = 1, \ldots, n_v$$

- n_h hypermultiplets

$$\{\zeta^{\mathcal{A}}, q^u\}, \qquad \begin{aligned} u &= 1, \ldots, 4n_h, \\ \mathcal{A} &= 1, \ldots, 2n_h \end{aligned}$$

Bosonic Lagrangian

$$\mathcal{L}_{\text{bos}} = \frac{R}{2} + \frac{1}{4}\,\text{Im}\,\mathcal{N}(\tau)_{\Lambda\Sigma}\,F^{\Lambda}_{\mu\nu}\,F^{\Sigma\mu\nu} - \frac{1}{8}\,\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}\,\text{Re}\,\mathcal{N}(\tau)_{\Lambda\Sigma}\,F^{\Lambda}_{\mu\nu}\,F^{\Sigma}_{\rho\sigma}$$
$$- g_{i\bar{\jmath}}(\tau)\,\partial_\mu\tau^i\partial^\mu\bar{\tau}^{\bar{\jmath}} - \frac{1}{2}\,h_{uv}(q)\,\mathrm{D}_\mu q^u \mathrm{D}^\mu q^v - V(\tau, q)$$

Electric and magnetic field strengths

$$F^\Lambda = \mathrm{d}A^\Lambda, \qquad \Lambda = 0, \ldots, n_v,$$
$$G_\Lambda = \star\left(\frac{\delta\mathcal{L}_{\text{bos}}}{\delta F^\Lambda}\right) = \text{Re}\,\mathcal{N}_{\Lambda\Sigma}\,F^\Lambda + \text{Im}\,\mathcal{N}_{\Lambda\Sigma}\,\star F^\Lambda$$

Scalar geometry

Non-linear sigma model: scalar fields = coordinates on target space

$$\mathcal{M} = \mathcal{M}_v(\tau^i) \times \mathcal{M}_h(q^u)$$

- $\mathcal{M}_v$ special Kähler manifold, $\dim_{\mathbb{R}} = 2n_v$, $\mathrm{U}(1)$ bundle
- $\mathcal{M}_h$ quaternionic manifold, $\dim_{\mathbb{R}} = 4n_h$, $\mathrm{SU}(2)$ bundle

Consequence of R-symmetry group $\mathrm{U}(2)_R = \mathrm{SU}(2)_R \times \mathrm{U}(1)_R$

Gaugings

Isometry group G (global symmetries) and local gauge group K

$$G \equiv \mathrm{ISO}(\mathcal{M}), \qquad K \subset G$$

Here $K = \mathrm{U}(1)^{n_v+1}$, two simpler possibilities:

- Fayet–Iliopoulos (FI): $n_h = 0$, ψ_μ^α charged under $\mathrm{U}(1) \subset \mathrm{SU}(2)_R$
- quaternionic gauging: Killing vectors k_Λ^u

$$k_\Lambda^u = \theta_\Lambda^{\mathcal{A}} k_{\mathcal{A}}^u, \qquad [k_\Lambda, k_\Sigma] = 0$$

$k_{\mathcal{A}}^u$ generates $\mathfrak{iso}(\mathcal{M}_h)$, $\theta_\Lambda^{\mathcal{A}}$ gauging parameters
$\mathcal{A} = 1, \ldots, \dim \mathrm{ISO}(\mathcal{M}_h)$

Symplectic covariance

- Field strength and Maxwell–Bianchi equations

$$\mathcal{F} = \begin{pmatrix} F^{\Lambda} \\ G_{\Lambda} \end{pmatrix}, \qquad \mathrm{d}\mathcal{F} = 0$$

Maxwell–Bianchi equations invariant under $\mathrm{Sp}(2n_v + 2, \mathbb{R})$

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- Section

$$\mathcal{V} = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \qquad \tau^i = \frac{L^i}{L^0},$$

- Maxwell charges

$$\mathcal{Q} = \frac{1}{\mathrm{Vol}\,\Sigma} \int_\Sigma \mathcal{F} = \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}$$

- Killing vectors, prepotentials and compensators

$$\mathcal{K}^u = \begin{pmatrix} k^{u\Lambda} \\ k^u_\Lambda \end{pmatrix}, \qquad \mathcal{P}^x = \mathcal{K}^u \omega^x_u + \mathcal{W}^x = \begin{pmatrix} P^{x\Lambda} \\ P^x_\Lambda \end{pmatrix}$$

  FI: $\mathcal{P}^3 = \mathrm{cst}$, EM charges of ψ^α_μ

- covariant formalism for BPS equation [1012.3756, Dall'Agata–Gnecchi]

Quartic function

Symplectic vector A: order-4 homogeneous polynomial

$$I_4 = I_4(A, \tau^i)$$

Define symmetric 4-tensor

$$t_{MNPQ} = \frac{\partial^4 I_4(A)}{\partial A^M \partial A^N \partial A^P \partial A^Q}$$

Different arguments and gradient

$$I_4(A, B, C, D) = t_{MNPQ} A^M B^N C^P D^Q$$
$$I_4'(A, B, C)^M = \Omega^{MR} t_{RNPQ} A^N B^P C^Q$$

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Quartic invariant

Symmetric space [hep-th/9210068, de Wit–Vanderseypen–Van Proeyen]

$$\partial_i I_4(A) = 0$$

C-map construction

- quaternionic manifold $\mathcal{M}_h$ built from special Kähler $\mathcal{M}_z$

$$q^u = \{\underbrace{\phi, \sigma, \xi^A, \xi_A}_{\text{fiber}}, \underbrace{Z^A, \bar{Z}_A}_{\mathcal{M}_z}\}$$

  $A = 1, \ldots, n_h$
- symplectic group $\mathrm{Sp}(2n_h, \mathbb{R})$
- symmetric $\mathcal{M}_z \to$ symmetric $\mathcal{M}_h$ – can use $\mathcal{I}_4$
- symplectic vectors

$$Z = \begin{pmatrix} Z^A \\ Z_A \end{pmatrix}, \qquad \xi = \begin{pmatrix} \xi^A \\ \xi_A \end{pmatrix}$$

Quaternionic Killing vectors

Isometries [hep-th/9210068, de Wit–Vanderseypen–Van Proeyen]

- universal symmetries: transformation of the fiber fields

$$\delta Z = 0, \qquad \mathcal{W}^x = 0$$

Computations: Killing vectors, prepotentials and compensators [1409.6310, H.E.–Halmagyi]

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- hidden symmetries: fiber-dependent $\mathcal{M}_z$ isometries

$$\delta Z = \mathbb{S}(\xi) Z, \quad \mathbb{S} = \frac{1}{2}\left(\xi \xi^t + \frac{1}{2}\, \mathbb{C}\partial_\xi \big(\mathbb{C}\partial_\xi \mathcal{I}_4(\xi)\big)^t\right) \mathbb{C} \in \mathfrak{sp}(2n_h, \mathbb{R})$$

Computations: Killing vectors, prepotentials and compensators [1409.6310, H.E.–Halmagyi]

AdS_4 vacua

- metric

$$\mathrm{d}s^2 = -\frac{r^2}{R^2}\,\mathrm{d}t^2 + \frac{R^2}{r^2}\,\mathrm{d}r^2 + \frac{r^2}{R^2}\,\mathrm{d}\Sigma_g^2$$

- BPS equations

$$\mathcal{P}^3 = -2\,\mathrm{Im}\,(\bar{\mathcal{L}}\mathcal{V}), \qquad \mathcal{L} = \frac{i\,\mathrm{e}^{i\psi_0}}{R}, \qquad \langle \mathcal{K}^u, \mathcal{V} \rangle = 0$$

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Contract last with ω_u^x (recall $\mathcal{P}^3 = \omega_u^3\mathcal{K}^u + \mathcal{W}^3$)

$$\mathcal{L} - \left\langle \mathcal{W}^3, \mathcal{V} \right\rangle = 0$$

$\mathcal{W}^3 = 0 \to$ no regular solution

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$$\mathcal{L} - \left\langle \mathcal{W}^3, \mathcal{V} \right\rangle = 0$$

$\mathcal{W}^3 = 0 \rightarrow$ no regular solution
Need non-trivial compensators from duality and hidden symmetries
$\rightarrow$ restriction on possible gaugings [1409.6310, H.E.–Halmagyi]

AdS–NUT black hole: Ansatz

Restrict to Fayet–Iliopoulos gauging

$$\mathcal{G} \equiv \mathcal{P}^3, \qquad \mathcal{P}^1 = \mathcal{P}^2 = 0$$

AdS–NUT dyonic black hole

$$\begin{aligned} \mathrm{d}s^2 &= -\,\mathrm{e}^{2U}\big(\mathrm{d}t + 2n\,H(\theta)\,\mathrm{d}\phi\big)^2 + \mathrm{e}^{-2U}\mathrm{d}r^2 + \mathrm{e}^{2(V-U)}\,\mathrm{d}\Sigma_g^2 \\ A^\Lambda &= \tilde{q}^\Lambda\big(\mathrm{d}t + 2n\,H(\theta)\,\mathrm{d}\phi\big) + \tilde{p}^\Lambda\,H(\theta)\,\mathrm{d}\phi \\ \tau^i &= \tau^i(r) \end{aligned}$$

Riemann surface Σ_g of genus g

$$\mathrm{d}\Sigma_g^2 = \mathrm{d}\theta^2 + H'(\theta)^2\,\mathrm{d}\phi^2, \qquad H(\theta) = \begin{cases} -\cos\theta & \kappa = 1 \\ \theta & \kappa = 0 \\ \cosh\theta & \kappa = -1 \end{cases}$$

with curvature $\kappa = \mathrm{sign}(1-g)$

NUT charge: preserves $\mathrm{SO}(3)$ isometry

AdS–NUT black hole: BPS equations

Define

$$\widetilde{\mathcal{V}} = \mathrm{e}^{V-U}\, \mathrm{e}^{-i\psi}\, \mathcal{V}$$

1/4-BPS equations [1503.04686, H.E.–Halmagyi]

– differential

$$2\, \mathrm{e}^{V} \partial_r \operatorname{Im} \widetilde{\mathcal{V}} = -\mathcal{Q} + I_4'(\mathcal{G}, \operatorname{Im} \widetilde{\mathcal{V}}, \operatorname{Im} \widetilde{\mathcal{V}}) + 2n\kappa \mathcal{G} r$$

$$(\mathrm{e}^{V})' = -2 \left\langle \operatorname{Im} \widetilde{\mathcal{V}}, \mathcal{G} \right\rangle$$

– algebraic

$$\mathrm{e}^{V} \left\langle \operatorname{Im} \widetilde{\mathcal{V}}, \partial_r \operatorname{Im} \widetilde{\mathcal{V}} \right\rangle = 2 \left\langle \operatorname{Im} \widetilde{\mathcal{V}}, \mathcal{Q} \right\rangle - 3n\kappa\, \mathrm{e}^{V} + 4n\kappa r \left\langle \mathcal{G}, \operatorname{Im} \widetilde{\mathcal{V}} \right\rangle$$

$$\langle \mathcal{Q}, \mathcal{G} \rangle = \kappa \in \mathbb{Z}$$

Note: BPS selects ± 1 for Dirac condition

- dynamical variables: only V and $\operatorname{Im} \widetilde{\mathcal{V}}$ appear
- $\mathcal{Q}$: integration constants from Maxwell equations

AdS–NUT black hole: BPS solutions

Ansatz

$$\mathrm{e}^{2V} = v_0 + v_1 r + v_2 r^2 + v_3 r^3 + v_4 r^4$$

$$\mathrm{Im}\,\widetilde{\mathcal{V}} = \mathrm{e}^{-V}\big(A_0 + A_1 r + A_2 r^2 + A_3 r^3\big)$$

V based on constant scalar solution and [Plebański–Demiański '76]

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Generic features

$$v_{p+1} = \frac{1}{p+1} \langle \mathcal{G}, A_p \rangle \,, \qquad p \geq 0$$

$$A_p = a_{p1}\,\mathcal{G} + a_{p2}\,\mathcal{Q} + a_{p3}\, I_4'(\mathcal{G}) + a_{p4}\, I_4'(\mathcal{G},\mathcal{G},\mathcal{Q}) + a_{p5}\, I_4'(\mathcal{G},\mathcal{Q},\mathcal{Q}) + a_{p6}\, I_4'(\mathcal{Q})$$

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Given $(\mathcal{G}, \mathcal{Q})$ and one constraint: analytic solution for symmetric space [1503.04686, H.E.–Halmagyi]

$$a_{pi} = a_{pi}(\mathcal{G}, \mathcal{Q}, n)$$

In particular

$$A_3 = \frac{1}{4}\,\frac{I_4'(\mathcal{G})}{\sqrt{I_4(\mathcal{G})}}, \qquad v_4 = \frac{1}{R^2_{\mathrm{adS}}} = \sqrt{I_4(\mathcal{G})}, \qquad S = \pi\sqrt{I_4(\mathrm{Im}\,\widetilde{\mathcal{V}})}\Big|_{r=r_h}$$

Outline: 4. Demiański–Janis–Newman algorithm

Introduction

Demiański–Janis–Newman algorithm [Newman–Janis '65] [Demiański–Newman '66] [Demiański '72]

- idea: **complex** change of coordinates $\to$ new charges (rotation, NUT)
- off-shell (derived metric is **not** necessarily solution)
- two prescriptions: Newman–Penrose formalism (more rigorous), direct complexification (quicker) [Giampieri '90] [1410.2602, H.E.]

Introduction

Demiański–Janis–Newman algorithm [Newman–Janis '65] [Demiański–Newman '66] [Demiański '72]

- idea: **complex** change of coordinates → new charges (rotation, NUT)
- off-shell (derived metric is **not** necessarily solution)
- two prescriptions: Newman–Penrose formalism (more rigorous), direct complexification (quicker) [Giampieri '90] [1410.2602, H.E.]
- main achievement: discovery of Kerr–Newman solution [Newman et al. '65]
- before 2014: defined only for the metric, 3 examples fully known (and 2 partly)
  (Kerr, BTZ, singly-rotating Myers-Perry)

Needs for supergravity

- gauge fields
- complex scalar fields
- topological horizons
- dyonic charges
- NUT charge: understand the complexification

Needs for supergravity

- ✓ gauge fields [1410.2602, H.E.]
- ✓ complex scalar fields [1501.02188, H.E.–Heurtier]
- ✓ topological horizons [1411.2909, H.E.]
- ✓ dyonic charges [1501.02188, H.E.–Heurtier]
- ✓ NUT charge: understand the complexification [1411.2909, H.E.]
- ✓ bonus: higher dimensions [1411.2030, H.E.–Heurtier]

Simple example (metric only)

Reissner–Nordström

$$\mathrm{d}s^2 = -f\,\mathrm{d}t^2 + f^{-1}\,\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \qquad f = 1 - \frac{2m}{r} + \frac{q^2}{r^2}$$

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Janis–Newman algorithm (Giampieri's prescription)

1) $\mathrm{d}t = \mathrm{d}u - f^{-1}\,\mathrm{d}r$

2) $u, r \in \mathbb{C}, \qquad f(r) \to \tilde{f} = \tilde{f}(r, \bar{r}) \in \mathbb{R}$

3) $u = u' + i\,j\cos\psi, \qquad r = r' - i\,j\cos\psi$

4) $i\,\mathrm{d}\psi = \sin\psi\,\mathrm{d}\phi, \qquad \psi = \theta$

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Janis–Newman algorithm (Giampieri's prescription)

$$\begin{aligned}
&1) \quad \mathrm{d}t = \mathrm{d}u - f^{-1}\,\mathrm{d}r \\
&2) \quad u, r \in \mathbb{C}, \qquad f(r) \to \tilde{f} = \tilde{f}(r, \bar{r}) \in \mathbb{R} \\
&3) \quad u = u' + i\,j\cos\psi, \qquad r = r' - i\,j\cos\psi \\
&4) \quad i\,\mathrm{d}\psi = \sin\psi\,\mathrm{d}\phi, \qquad \psi = \theta
\end{aligned}$$

Kerr–Newman (Boyer–Lindquist coordinates)

$$\mathrm{d}s^2 = -\tilde{f}\,\mathrm{d}t^2 + \frac{\rho^2}{\Delta}\mathrm{d}r^2 + \rho^2\mathrm{d}\theta^2 + \frac{\Sigma^2}{\rho^2}\sin^2\theta\,\mathrm{d}\phi^2 + 2j(\tilde{f} - 1)\sin^2\theta\;\mathrm{d}t\mathrm{d}\phi$$

$$\tilde{f} = 1 - \frac{2m\,\mathrm{Re}\,r}{|r|^2} + \frac{q^2}{|r|^2} = 1 - \frac{2mr - q^2}{\rho^2}, \qquad \rho^2 = r'^2 + j^2\cos^2\theta$$

Keys ingredients

- gauge field: gauge transformation to set $A_r = 0$
  $\rightarrow$ missing step in [Newman et al. '65]!
  (other approach: [1407.4478, Keane])

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$$Z = q + ip$$

  instead of q and p
- adding a NUT charge: complexify the mass, shift horizon curvature

$$m = m' + i\kappa n, \qquad \kappa = \kappa' - \frac{4\Lambda}{3}\, n^2.$$

New examples

- Kerr–Newman–NUT
- dyonic Kerr–Newman
- Yang–Mills Kerr–Newman
- adS–NUT Schwarzschild
- BPS solutions from $N = 2$ ungauged supergravity
- (Sen's) non-extremal rotating black hole in T^3 model
- SWIP solutions
- charged Taub–NUT–BBMB with Λ
- $5d$ Myers–Perry
- BMPV

Outline: 5. Conclusion

AdS–NUT black holes

Demiański–Janis–Newman algorithm:

- (almost) all examples can be embedded in $N = 2$ supergravity
- non-extremal adS–NUT black hole in gauged $N = 2$ sugra with $F = -i\, X^0 X^1$ [Klemm–Rabbiosi, private communication]
- consequence of supersymmetry / U-duality / string theory?
- derive 1/4-BPS black holes with $n \neq 0$ from the ones with $n = 0$?

Achievements

- symplectic covariant quaternionic Killing vectors (and derived quantities)
- conditions for $N = 2$ adS_4 vacua and near horizon-geometries $\mathrm{adS}_2 \times \Sigma_g$
- general analytic solution of 1/4-BPS dyonic adS–NUT black holes with running scalars in $N = 2$ FI supergravity
- extend Demiański–Janis–Newman algorithm, in particular to supergravity

Outlook

- Demiański–Janis–Newman algorithm
  - more $N = 2$ gauged supergravity solutions
  - $d \geq 6$ Myers–Perry
  - multicenter solutions
  - black rings
- $1/2$-BPS adS–NUT black holes
- BPS solutions with rotation and acceleration

Thank you!

Merci !